L^2-Gradient Flows of Spectral Functionals

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Abstract

The study of gradient flows for spectral shape functionals is a difficult issue because of both the choice of the metric driving the evolution and the non-convexity of the functionals involved. In this talk, we consider the L^2-gradient flow of functionals $\mathcal{F}$ depending on the eigenvalues of Schrödinger potentials V for a wide class of differential operators associated to closed, symmetric, and coercive bilinear forms, including the case of all the Dirichlet forms (as for second order elliptic operators in Euclidean domains or Riemannian manifolds). In this way, we have a relatively simple metric driving the evolution, but we still have to deal with non-convex spectral functionals.

Precisely, we suppose that $\mathcal{F}$ arises as the sum of a $-\theta$-convex functional $\mathcal{K}$ forcing the admissible potentials to stay above a constant $V_{\min}$ and a term $\mathcal{H}(V) = \varphi(\lambda_1(V), \cdots, \lambda_J(V))$ which depends on the first J eigenvalues associated to V through a C^1 function φ.

Even if $\mathcal{H}$ is not a smooth perturbation of a convex functional (and it is in fact concave in simple important cases as the sum of the first J eigenvalues) and we do not assume any compactness of the sublevels of $\mathcal{K}$, we prove the convergence of the Minimizing Movement method to a solution $V \in H^1(0, T; L^2)$ of the differential inclusion $V'(t) \in -\partial_L^- \mathcal{F}(V(t))$, which under suitable compatibility conditions on φ can be written as

$$V'(t) + \sum_{i=1}^{J} \partial_i \varphi(\lambda_1(V(t)), \dots, \lambda_J(V(t))) u_i^2(t) \in -\partial_F^- \mathcal{K}(V(t))$$

where $(u_1(t), \dots, u_J(t))$ is an orthonormal system of eigenfunctions associated to the eigenvalues $(\lambda_1(V(t)), , \dots, \lambda_J(V(t)))$.

This is a joint work with G. Savaré (Bocconi).