Boundary conditions and inf-sup condition in the discretization of elliptic problems by neural networks

Claudio Canuto
Politecnico di Torino

Abstract

Variational Physics-Informed Neural Networks (VPINN) are an instance of application of deep learning to the solution of boundary-value problems. In VPINN, the network is trained by minimizing a functional of the weak residual of the problem. We look at VPINN from a Petrov-Galerkin perspective, where the spaces of test functions are built by finite elements. We discuss how to enforce essential boundary conditions, and we introduce a variant of VPINN, named IVPINN, which incorporates a piecewise polynomial interpolation of the neural network. This guarantees the numerical stability of the solution via the fulfillment of an inf-sup condition, and yields a priori and a posteriori error estimates in the energy norm. Numerical experiments illustrate the performances of the method.

This is a joint work with Stefano Berrone, Moreno Pintore, and N. Sukumar.