Variations on the Erdős distinct-sums problem

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Abstract

Let $\{a_1, ..., a_n\}$ be a set of positive integers with $a_1 < \ldots < a_n$ such that all 2^n subset sums are distinct. A famous conjecture by Erdős states that $a_n > c \cdot 2^n$ for some constant c, while the best result known to date is of the form $a_n > c \cdot 2^n/\sqrt{n}$. In this talk, we give an overview on the different methods that have been used, during the past years, to provide some nontrivial lower bounds on a_n (see [2, 3, 4]). Then, inspired by an information-theoretic interpretation, in [1], we extend the study to vector-valued elements $a_i \in \mathbb{Z}^k$ and we weaken the condition by requiring that only sums corresponding to subsets of size smaller than or equal to λn be distinct. For this case, we derive lower and upper bounds on the smallest possible value of a_n.

Keywords: Erdős distinct-sums problem, polynomial method, probabilistic method
MSC: 05D40, 11B13

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