

# Cwikel's Bound Reloaded

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**Abstract:** There are a couple of proofs by now for the famous Cwikel-Lieb-Rozenblum (CLR) bound, which is a semiclassical bound on the number of bound states for a Schroedinger operator, proven in the 1970s. Of the rather distinct proofs by Cwikel, Lieb, and Rozenblum, the one by Lieb gives the best constant, the one by Rozenblum does not seem to yield any reasonable estimate for the constants, and Cwikel's proof is said to give a constant which is at least about 2 orders of magnitude off the truth. This situation did not change much during the last 40+ years.

It turns out that this common belief, i.e., Cwikel's approach yields bad constants, is not set in stone: We give a drastic simplification of Cwikel's original approach which leads to an astonishingly good bound for the constant in the CLR inequality. Our proof is also quite flexible and leads to rather precise bounds for a large class of Schroedinger-type operators with generalized kinetic energies. Moreover, it highlights a natural but overlooked connection of the CLR bound with bounds for maximal Fourier multipliers from harmonic analysis.