Sharp functional inequalities via Optimal Mass Transport Theory

Sobolev and isoperimetric type inequalities find many applications in PDE and Calculus of Variations. A classical technique to obtain some of these inequalities both in Euclidean and curved setting is the symmetrization. More recently, Cordero Erasquin, Nazaret and Villani made use of the Optimal Mass Transport Theory (OMT) to prove sharp Sobolev and Gagliardo Nirenberg inequalities. With this approach, the study of the equality case becomes notably easier. We use OMT to show a generalized version of the Gagliardo-Nirenberg inequality containing three homogeneous weights satisfying a suitable concavity condition. As application, we find weighted sharp p-log-Sobolev, isoperimetric and Faber-Krahn inequalities with explicit sharp constants. With an independent proof, we also characterize all the extremals for the weighted p-log-Sobolev inequality, for any p greater than or equal to 1. This characterization is new also in the unweighted case for p larger than the dimension of the space. The talk is based on two papers made in collaboration with Z. Balogh (Bern) and A. Kristály (Cluj-Napoca/Budapest).

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