YoungPeople4Math 2025

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Room 1, Università Cattolica del Sacro Cuore,
via Garzetta 48, Brescia

Organized by Seminario Matematico di Brescia

Università degli Studi di Brescia   Università Cattolica del Sacro Cuore

Programme

- 09:30-09:40 – *Welcome*
- 09:40-10:10 – **Marco Gallo**: It's a trap! A concavity trap!
- 10:15-10:45 – **Santo Saraceno**: Renewable Energy Communities with Peer-to-Peer Exchange: a Chance-Constraint Approach
- 10:45-11:10 – *Coffee break*
- 11:10-11:40 – **Davide Mattiolo**: r-Graphs with edges in exactly one perfect matching
- 11:45-12:15 – **Erika Temellini**: A Hierarchical Model Reduction approach for fluid dynamics applications
- 12:15-14:00 – *Lunch break*
- 14:00-14:30 – **Alessandro Gobbi**: A Stochastic Programming approach for combined forward and reverse logistics in hub-and-spoke e-commerce networks
- 14:35-15:05 – **Gabriele Barbieri**: Diffeology and Symplectic Reduction
- 15:10-15:40 – **Paolo Tesini**: The Spline Upwind Method for Space–Time Isogeometric Analysis and Applications in Cardiac Electrophysiology
- 15:40-16:05 – *Coffee break*
- 16:05-16:35 – **Silvia Paparini**: A theoretical framework for shape instabilities in liquid crystalline network sheets with central topological defects
- 16:40-17:10 – **Stefano Della Fiore**: An overview of the Graham's rearrangement conjecture

Abstracts

Diffeology and Symplectic Reduction

Gabriele Barbieri (Università di Milano-Bicocca e Università di Pavia)

Diffeology, introduced in the 1980s by J.-M. Souriau and his students, provides a flexible framework for differential geometry on spaces that may be singular or infinite-dimensional–such as irrational tori, symplectic orbifolds, or groups of symplectomorphisms. Instead of building on charts like manifolds do, diffeology defines smoothness through a collection of parametrizations from Euclidean spaces that satisfy a few natural conditions. Manifolds themselves fit naturally into this framework as a special case, making diffeology an extension of classical differential geometry.

In the first part of the talk, I will introduce the basic ideas and definitions of diffeology, along with illustrative examples. In the second part, I will present a recent result, developed in joint work with J. Watts and F. Ziegler, concerning an application of diffeology to symplectic reduction. Specifically, we study a symplectic version of the representation-theoretical Frobenius Reciprocity, introduced by V. Guillemin and S. Sternberg and refined by T. Ratiu and F. Ziegler, which establishes a bijection between certain reduced spaces. These spaces are not necessarily manifolds, but with diffeology, one can still make sense of smooth maps and differential forms on them. We show that this correspondence is indeed smooth in the diffeological sense, and provide new criteria for the existence of reduced symplectic forms–such as when the group action is proper or locally free.

An overview of the Graham's rearrangement conjecture

Stefano Della Fiore (Università degli Studi di Brescia)

A famous conjecture of Graham stated in 1971 asserts that for any set $A \subseteq \mathbb{Z}_p \setminus \{0\}$ there is an ordering $a_1, \ldots, a_{|A|}$ of the elements of A such that the partial sums $a_1, a_1 + a_2, \ldots, a_1 + a_2 + \ldots + a_{|A|}$ are all distinct. Before the recently improvements, the state of the art was essentially that the conjecture holds when $|A| \leq 12$ and when A is a non-zero sum set of size $p-1$, $p-2$ or $p-3$ (see [1] and the references therein). Many of the arguments for small A use the Polynomial Method and rely on Alon's Combinatorial Nullstellensatz (see [2]). Very recently, Kravitz in [3], using a rectification argument, made a significant progress proving that the conjecture holds whenever $|A| \leq \log p / \log \log p$. A subsequent paper of Bedert and Kravitz [4] improved the logarithmic bound into a super-logarithmic one that is of the form $e^{c(\log p)^{1/4}}$ for some small constant $c > 0$. Finally, Costa, Della Fiore and Engel [5] revisiting their method improved their bound to $e^{c(\log p)^{1/2}}$.

In this talk, we give an overview of the methods and tools used to provide such upper bounds.

[1] S. Costa, F. Morini, A. Pasotti, M. A. Pellegrini. A problem on partial sums in abelian groups. *Discrete Mathematics*, 341 (2018).

[2] S. Costa, S. Della Fiore, M. A. Ollis and S. Z. Rovner-Frydman. On Sequences in Cyclic Groups with Distinct Partial Sums, *Electron. J. Combin.*, 29 (2022).

[3] N. Kravitz. Rearranging small sets for distinct partial sums, *Integers: Electronic Journal of Combinatorial Number Theory*, 24 (2024).

[4] B. Bedert, N. Kravitz. Graham's rearrangement conjecture beyond the rectification barrier, *to appear in Israel J. Math.*, (2025).

[5] S. Costa, S. Della Fiore and E. Engel. New results on the Graham's rearrangement conjecture, *in preparation*, (2025).

It's a trap! A concavity trap!

Marco Gallo (Università Cattolica del Sacro Cuore)

Differential equations is a common topic that arise in several fields; besides existence and uniqueness, people are often interested in some qualitative properties of the solutions. Considering a positive solution u which is asked to be zero on the boundary of some convex domain $\Omega \subset \mathbb{R}^N$,

$$-\Delta u = f(u) \ \text{ in } \Omega, \quad u > 0 \ \text{ in } \Omega, \quad u = 0 \ \text{ on } \partial\Omega,$$

it is natural – and often useful – to investigate if such a solution is concave or not. In this talk, we will discuss when this happens, and what we can do when it does not. Eventually, we will deal also with some evolution in time ∂_t, and see how to suitably "trap" our solutions.

Some of the results presented are in collaboration with Riccardo Moraschi and Marco Squassina.

A Stochastic Programming approach for combined forward and reverse logistics in hub-and-spoke e-commerce networks

Alessandro Gobbi (Università della Calabria)

A robust strategy to managing e-commerce logistics integrates forward and reverse logistics systems, ensuring the collection of returns alongside traditional product distribution. This technique employs hub-and-spoke networks to aggregate both distribution and collection demands from several customers into a few central hubs. Within this framework, we examine a complex variant of the Vehicle Routing Problem with divisible deliveries and pickups, where each hub may have mandatory delivery and return pickup demands and can be visited multiple times within the same or different routes [1]. Due to the large fluctuation of demand within the aggregating hubs, we also assume that an uncertain optional pickup quantity may arise. The objective of the problem is to minimize the overall cost by strategically routing a homogeneous fleet of vehicles that fulfills mandatory deliveries and pickups while ensuring that a minimum percentage of optional pickups is also met. By doing so, we prevent an excessive accumulation of pickup requests for future days. We propose a two-stage Stochastic Programming formulation, including ad-hoc recourse actions [2], approximated through a scenario-based deterministic equivalent approach. We conducted several experiments to validate the mathematical model by using different probability distributions to simulate the variation of optional requests. Finally, we present some managerial insights useful to wisely face the impact of uncertainty in similar settings.

[1] G. Nagy, N.A. Wassan, M.G. Speranza, C. Archetti. The vehicle routing problem with divisible deliveries and pickups. *Transportation Science*, 49(2), 271-294 (2015).

[2] M. Salavati-Khoshghalb, M. Gendreau, O. Jabali, W. Rei. A Rule-Based Recourse for the Vehicle Routing Problem with Stochastic Demands, *Transportation Science* 53(5), 1334-1353 (2019).

r-Graphs with edges in exactly one perfect matching

Davide Mattiolo (KU Leuven)

Let r be a positive integer. A graph is *r-regular* if each of its vertices is incident with exactly r edges. An *r-graph* is an r-regular graph G such that every odd set of vertices $X \subseteq V(G)$ is connected by at least r edges to its complement $V(G) \setminus X$.

A well-known theorem of Petersen [1] claims that every 3-graph has a perfect matching. A strengthening of Petersen's result, due to Schönberger [2], states that every edge of a 3-graph G is contained in at least one perfect matching of G. In fact, the same property can be deduced for every r-graph. In light of this fact, it is natural to ask whether one can characterize those r-graphs having edges, called *lonely*, which belong to exactly one perfect matching.

In this talk we first observe that, in order to study lonely edges in r-graphs, one may restrict to 3-edge-connected r-graphs. Then, we present structural characterizations of some 3-edge-connected r-graphs having lonely edges.

The talk is based on a joint work with J. Goedgebeur, G. Mazzuoccolo, J. Renders and I. H. Wolf, and a joint work with D. V. V. Narayana, K. Gohokar and N. Kothari.

[1] J. Petersen, Die theorie der regulären graphen, *Acta Math.* 15, 193–200, (1891).

[2] T. Schönberger, Ein beweis des petersenschen graphensatzes, *Acta Sci. Math. (Szeged)* 7, 51–57, (1934).

A theoretical framework for shape instabilities in liquid crystalline network sheets with central topological defects

Silvia Paparini (Università degli Studi di Padova)

Liquid crystalline networks (LCNs) are stimuli-responsive materials formed from polymeric chains cross-linked with rod-like mesogenic segments, which, in the nematic phase, align along a non-polar director. A key characteristic of these nematic systems is the existence of singularities in the director field, known as topological defects or disclinations, and classified by their topological charge. In this talk, we address the open question of modeling mathematically the coupling between mesogens disclination and polymeric network by providing a mathematical framework describing the out-of-plane shape changes of initially flat LCN sheets containing a central topological defect. Adopting a variational approach, we define an energy associated with the deformations consisting of two contributions: an elastic energy term accounting for spatial director variations, and a strain-energy function describing the elastic response of the polymer network. The interplay between nematic elasticity, which seeks to minimize distortions in the director field, variations in the degree of order, with the consequent tendency of monomers in the polymer chains to distribute anisotropically in response to an external stimulus, and mechanical stiffness, which resists deformation, determines the resulting morphology. We analyze the transition to instability of the ground-state flat configuration and characterize the corresponding buckling modes.

This is a joint work with Giulio G. Giusteri and L. Angela Mihai.

Renewable Energy Communities with Peer-to-Peer Exchange: a Chance-Constraint Approach

Santo Saraceno (Università degli Studi di Brescia)

This work presents a chance-constraint model for the management of Energy Communities, focusing on prosumers and peer-to-peer electricity exchanges. The model aims to minimize the total operation costs of the community, while ensuring energy balance and satisfying technical constraints related to local production and the energy exchanges both inside the community and with the main grid.

Uncertainty in solar photovoltaic generation and electricity demand is addressed using individual and joint chance constraints that are modeled using normal distributions and approximated through piecewise-linear techniques when necessary.

The model is tested on a prototype example of community and is implemented in Python using Pyomo.

A Hierarchical Model Reduction approach for fluid dynamics applications

Erika Temellini (Politecnico di Milano)

Hierarchical Model (HiMod) reduction is a methodology designed to tackle problems characterized by a dominant directionality. These problems are commonly encountered in real-world

scenarios such as hemodynamics, acoustic wave propagation, and industrial circuit or pipeline optimization. Despite the main directionality, these problems remain inherently three-dimensional, resulting in computationally unaffordable simulations.
HiMod exploits the separation of variables principle by discretizing the dynamics along the main flow direction via Finite Element methodology (FEM), while employing a modal expansion to approximate the transverse dynamics. This approach reduces the original full-order (3D or 2D) problem to a system of coupled 1D equations [1], significantly reducing the computational cost while maintaining a high level of accuracy, as demonstrated in various application fields [2, 3, 4]. In this communication, we first provide the fundamental background to understand the core features of HiMod methodology. Then, we present the latest advancements, specifically in the context of the Stokes equations for fluid dynamics modeling. On this line, we introduce the key concepts that enable a rigorous well-posedness analysis of the HiMod formulation for the Stokes problem. Successively, we propose a specific set of modal basis functions for fluid dynamics applications, computed as the solution to a tailored Sturm-Liouville eigenvalue problem [5]. Owing to the robustness of the methodology and the promising performance of the newly developed modal basis functions – although still currently on validation – HiMod reduction emerges as an ideal tool for addressing a broad spectrum of future engineering applications.

This is joint work with F. Ballarin, T. Chacon Rebollo and S. Perotto.

[1] S. Perotto, A. Ern, and A. Veneziani, Hierarchical local model reduction for elliptic problems: a domain decomposition approach, *Multiscale Model. Simul.* 8(4), 1102–1127 (2010).

[2] Y. A. Brandes Costa Barbosa and S. Perotto, Hierarchically reduced models for the Stokes problem in patient-specific artery segments, *Int. J. Comput. Fluid Dyn.* 34(2), 160–171 (2020).

[3] G. G. Gentili, M. Khosronejad, G. Bernasconi, S. Perotto, and S. Micheletti, Efficient modeling of multimode guided acoustic wave propagation in deformed pipelines by hierarchical model reduction, *Appl. Numer. Math.* 173, 329–344 (2022).

[4] G. Conni, S. Piccardo, S. Perotto, G. M. Porta, and M. Icardi, HiPhomε: High-Order Projection-Based Homogenization for Advection-Diffusion-Reaction Problems, *Multiscale Model. Simul.* 23(1), 640–667 (2025).

[5] M. C. Aletti, S. Perotto, and A. Veneziani, HiMod reduction of advection-diffusion-reaction problems with general boundary conditions, *J. Sci. Comput.* 76(1), 89–119 (2018).

The Spline Upwind Method for Space–Time Isogeometric Analysis and Applications in Cardiac Electrophysiology

Paolo Tesini (Università degli Studi di Brescia)

This work introduces the Spline Upwind (SU) method, a novel stabilization technique for solving the heat equation within the space-time Isogeometric Analysis (IgA) framework, employing simultaneous discretization in both space and time. The SU method extends the Streamline Upwind Petrov-Galerkin (SUPG) stabilization to high-degree, high-continuity splines, ensuring temporal causality and mitigating spurious oscillations in numerical solutions. Furthermore, a modified version of the method is proposed for applications in cardiac electrophysiology. We validate the proposed method through a series of numerical experiments in 1D, 2D and 3D spatial domains, corresponding to 2D, 3D and 4D space–time domains. To handle the computational complexity of 4D simulations, on domains approximating left ventricular geometry, we employ efficient solvers with preconditioning techniques. Numerical results present optimal convergence order for smooth solutions and stability in the presence of sharp layers and the SU method thus provides a robust and computationally efficient approach, ensuring both numerical stability and cost-effectiveness.