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A thermodynamic non-equilibrium model for the expansion of a real gas in a turbine stage

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ABSTRACT: Exergy is a fundamental thermodynamic property in the field of energy engineering: it is defined as “*the maximum useful work that can be extracted from a system in a state S_1 as it comes into equilibrium (at S_0) with a reference environment O , while interacting only with said environment*”. Unlike energy, which is conserved, **exergy is not conserved, but rather** destroyed by irreversibilities in every real process like friction, heat transfer through a finite temperature difference, mixing, etc.

This lecture describes the problem of the evolution of systems from a given initial state to a final one in the most general case in which the transformations include non-equilibrium states. The chosen example is the expansion of a real gas in a turbine stage (nozzle + rotor). The model is based on an equation of motion derived from the real Ginzburg-Landau equation but rephrased in terms of exergy. The fundamental assumption here is that the evolution of the fluid is driven by the specified temperature and pressure differences between the up- and downstream boundaries, and that the details of the intermediate states are linked to the local blade deflection angle β . Thus, for a fixed initial pair (T_{Ti}, p_{Ti}) and an assigned degree of reaction, it is possible to explicitly express the local work, friction- and heat losses along the passage as functions of the cascade head- and flow coefficients, ψ and ξ respectively. The model does not make use of the Local Equilibrium Hypothesis (LEH), and for simplicity's sake in the example discussed here a quasi-1D approach is adopted, assuming that at each station along its path the fluid is homogeneous in the directions perpendicular to the main motion (radial velocity identically zero and tangential velocity constant in the circumferential direction).

The model calculates the (transversally averaged) non-equilibrium exergy at each station along the chord, and the main result is that its value at rotor exit is substantially higher than its equilibrium counterpart. The evolution history depends strongly on the deflection angle $\Delta\beta$, i.e. on both the head- and flow coefficients. The solution to the mass- and energy balances leads to an analytical expression for the non-equilibrium exergy, from which a “non-equilibrium Temperature” can be formally derived.

The paradigm is theoretically simple and the resulting model of relative ease of implementation (the solution presented here was obtained on a I5 core using MATHEMATICA), and fully two-dimensional solutions may be obtained as well, provided a proper form of the heat equation is used to calculate the fluid-to-wall thermal diffusion. Applications of the proposed framework may help designers to gain a better insight into real non-equilibrium expansion processes and to more accurately tune the nozzle- and rotor efficiency.

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