TAX EVASION AND THE COST OF PUBLIC SECTOR ACTIVITIES

by
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Tax evasion and the cost of public sector activities *

Rosella Levaggi

ABSTRACT

This paper examines the effects that tax evasion has on the cost to produce goods and services in an environment where input prices are homogenously taxes, but evasion is intersectorially differentiated. Tax evasion raises the relative cost of producing goods and services in the sectors where evasion is more difficult. Such effects adds a potentially important, but so far neglected, element to the relative cost of goods and services produced by the public sector. The general equilibrium analysis presented in this paper shows the perverse effects of tax evasion which reduces total production (hence total wealth) even in a model where, due to the absence of redistributive effects, tax evasion is simply fiscal illusion.

J.E.L. : H50, H26

Keywords: Marginal cost of public funds, tax evasion, general equilibrium, Baumol’s cost disease

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1. Introduction

The effect of taxation and tax evasion has been widely studied by the literature, but there does not seem to be a generalised consensus on optimal tax strategies and the role of tax evasion in the economy.

The traditional and long established literature on optimal tax design suggests to choose instruments that alter as little as possible the behaviour of taxpayers. In this way, the deadweight loss from taxation is minimized. In this environment, tax evasion is seen as a side effect of taxation that should be avoided as much as possible since a parallel and increasing literature shows its perverse effects on the economy.

This view is not shared by a part of the most recent literature. Richter and Boadway (2005), for example, argue that an optimal tax design should take account that different taxes are evaded to a different extent and that the principles of efficient tax design must be revised to take account of this differential ease with which taxes can be evaded.

Rosen (2005) argue that tax evasion might even be welfare improving by allowing some economic activities to be taxed at lower rates than others, in a way that is consistent with optimal tax rule. Davidson, Martin and Wilson (2005) build on this intuition and show that under special conditions tax evasion replicates an optimal discriminatory tax system.

In this note we want to study an aspect of taxation and tax evasion, so far neglected by the literature, that relates to the effects of the latter on the relative cost of inputs in sectors that are characterized by a different level of tax evasion. In particular we want to study the effects on the relative cost of goods and services produced by the public sector in an environment where tax evasion is possible, but only in the private sector.

We argue that tax evasion produces the same effects as a discriminatory tax on a good (labour in our case) which is homogeneous and causes an increase in the relative price of the good produced by the sector where evasion is not possible. An alternative way to interpret this result is in term of marginal cost of the public funds; the reaction to taxation produces a series of actions which would be harmful, counterproductive or wasteful in the absence of taxation, but which are personally advantageous because they reduce (or seem to) the tax bill. The literature has studied some of these behaviours, usually with reference to adjustments of consumers in their labour supply or demand choices. This article adds a new dimension to this study by proposing a further effect deriving from tax evasion and affecting directly the production side.

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1 For review of the policy issues concerning tax evasion see Sandmo (2005)
2 For a review see Slemrod and Yitzahi (2002)
The rising cost of goods produced in the public sector and tax evasion have not been widely studied together, and this in spite of hard and casual evidence that there is less tax evasion associated with public sector inputs, both in terms of factor inputs and intermediate inputs purchased from other sectors. This result, obtained in a general equilibrium framework, is independent from the organisation of production, i.e. the effect on the relative cost is independent from the degree of centralisation of Government decisions and the way public sector production is financed. The perverse effects that tax evasion has on tax revenue and the distribution of the fiscal burden are well understood. Cost adds a new dimension to the effects of tax evasion, so far neglected in the literature, which should be accounted for when assessing the cost and benefits of tax-financed public provision and when designing tax compliance incentives.

The remainder of the paper is organised as follows: section 2 presents some evidences; section 3 presents the model and section 4 concludes.

2. Tax evasion and the cost of the public sector

Tax evasion in the public sector is less widespread than in the private one for several well-known reasons. Goods and services have to be fully invoiced and workers are employed through a regular contract. Incomes deriving from public sector activities are difficult to be hidden from the tax administration owing to their prevalent taxing method (source method) and the higher probability of being caught (cross checks are quite simple).

The increase in tax rates and the social security contribution burden is the main cause for the increase in the shadow economy\(^4\) as shown by the most recent empirical literature on the subject.\(^5\) Schneider (2003, 2005) shows that the shadow economy, a good proxy for tax evasion, is increasing in OECD and transition economies (from 13.2% in 1990 to 16.7% in 2001 in OECD countries).

Also the relative price for public production, defined as the ratio of the implicit price for government consumption and the same indicator for household consumption, is well over one and tends to increase through time. Baumol (1967) and the subsequent literature proposes the following explanation: productivity in the public sector grows at a slower pace than in the private sector while wages grow at the same pace, hence the relative cost increases. However,

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\(^3\) See Browning (1976), Stuart (1985), Usher (1986), Ballard and Fullerton (1992)

\(^4\) Also called the underground, informal, or parallel economy, the shadow economy includes not only illegal activities but also unreported income from the production of legal goods and services, either from monetary or barter transactions. Hence, the shadow economy comprises all economic activities that would generally be taxable were they reported to the tax authorities.
there is mixed evidence on this relationship; while in some services, such as education, productivity has been lower than in the rest of the economy (Gundlach et al., 2001), for the service industry in general it is not always true that its level of productivity is lower than the manufacturing sector (Schreyer and Pilat, 2001). The increase in the relative price of the public sector might be caused by other factors, perhaps in addition to lower productivity growth.

Figure one shows the existence of a positive relationship between the relative price for public output and the size of the shadow economy (as % of GPD) for OECD countries in 2001/02.

![Image](image.png)

**Fig.1: Cross-country comparison**

The relative price for public output is the ratio of the implicit price for government consumption by the implicit price for household consumption using OECD data. For tax evasion, the proxy used is the size of the shadow economy using the currency demand approach (Schneider, 2003\(^5\), 2005) as a percent of GDP. The relationship between the two variables is positive. Countries such as Switzerland and the US where the shadow economy is relatively small (around 8.5%) present a relative price close to one. When tax evasion increases, the price goes up considerably, as in the case of Greece where the shadow economy

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\(^5\) See Schneider (2003, 2005) for a review.

\(^6\) The data used are derived from the same paper.
represents about 28.5% of GDP and the relative price for public output is around 1.25. This relationship is robust to changes in the size of tax evasion in both directions, as shown in table 1 where the available data for the shadow economy of a low tax evasion country (US) and of an high one (Italy) are recorded.

<table>
<thead>
<tr>
<th>Year</th>
<th>US</th>
<th></th>
<th>Italy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in relative price</td>
<td>Change in shadow economy</td>
<td>Change in relative price</td>
<td>Change in shadow economy</td>
</tr>
<tr>
<td>1995</td>
<td>-0.292%</td>
<td>-4.255%</td>
<td>0.833%</td>
<td>1.550%</td>
</tr>
<tr>
<td>1996</td>
<td>-0.293%</td>
<td>-4.444%</td>
<td>0.826%</td>
<td>3.053%</td>
</tr>
<tr>
<td>1997</td>
<td>0.588%</td>
<td>2.326%</td>
<td>1.639%</td>
<td>1.111%</td>
</tr>
<tr>
<td>1998</td>
<td>0.682%</td>
<td>1.136%</td>
<td>0.806%</td>
<td>0%</td>
</tr>
<tr>
<td>1999/2000</td>
<td>-0.677%</td>
<td>-2.247%</td>
<td>0.800%</td>
<td>-0.733%</td>
</tr>
<tr>
<td>2001/2002</td>
<td>-0.292%</td>
<td>0%</td>
<td>-0.794%</td>
<td>-0.369%</td>
</tr>
</tbody>
</table>

Tab. 1: Evolution through time

Although a formal test cannot be performed due to the lack of available data, the evidence points towards the existence of a positive relationship between the relative price of goods produced by the public sector and the relative size of tax evasion. This positive relationship can be interpreted in terms of an excess burden created by tax evasion.

3. The Model

A two-sector economy is considered. A competitive sector is producing an appropriable good and the other sector produces a public good by means of a single output, production function whose inputs are represented by labour and capital. Both inputs are supplied in fixed quantities so that in this context any tax on the use of these factors should be a lump-sum since it does not alter the supply of such factors. The demand side is very stylized. We assume there is only one consumer who has an initial endowment of labour and capital that is supplied to firms to produce two goods. One of them is a non-marketed good whose production cost is paid in the form of an income tax. For the purpose of the analysis presented here, the demand side, as well as the financial part does not play any role: the effects of tax evasion will be studied in the production sector.

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7 A more formalised model to test this hypothesis cannot be implemented due to lack of available data on the size of the shadow economy.

8 Public good is used here in a broad sense and it means a composite good that is produced by the public sector.
3.1 The production side

Produced goods are denoted $X$ and $Y$. The primary factors are labour ($L$) and capital ($K$). There are two productive sectors or industries, denoted $S_X$ and $S_Y$, producing $X$ and $Y$; both goods are produced using a production function homogeneous of degree one:

$$X = f^X(K^X, L^X)$$
$$Y = f^Y(K^Y, L^Y).$$

(1)

All the markets are in equilibrium and there is full employment in the economy, i.e.:

$$L^X + L^Y = L$$
$$K^X + K^Y = K$$

(2)

$Y$, the public good, is financed through a uniform income tax at rate $t$ on capital and labour earnings. The production of $Y$ can be organised in two different ways:

a) a centralised system where the agency that raises taxes also produces the public good;

b) a decentralised system where the good is produced by an agency that receives a grant to produce it.

Given that the production factors are intersectorally mobile and have to be fully employed, after-tax wages and the price for capital are equal in the two sectors, an assumption common to the literature dealing with unbalanced growth, the unit prices are equal to $w$ and $r$ respectively.

The organisation of production in the public sector might have important consequences for the perceived price of the production factors. In a centralised system, the agency will perceive the price net of taxes since it pays income to itself. In a decentralised system, this might not be the case since taxes are a true cost for the agency.

For a centralised system, the conditional factor demand for the two industries can be written as:

$$w = f^X_{L^X}$$
$$r = f^X_{K^X}$$
$$w(1-t) = f^Y_{L^X}$$
$$r(1-t) = f^Y_{K^X}$$

(3)

where superscripts denote production function and the subscripts first derivatives. For a decentralised system we can write:

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9 The use of a wage market clearing mechanism can be justified in a variety of ways ranging from the explanation provided by Baumol, who saw it as the effect of organised labour, to Pasinetti and Sraffa who explain this condition in terms of uniform quality of labour in the two sectors (see Notarangelo 1999).
\[ w = f_{LX}^X \]
\[ r = f_{KX}^X \]
\[ w = f_{LX}^Y \]
\[ r = f_{KX}^Y \]  
\[ \text{(4)} \]

Using (3/4), both expressions can be written as:
\[ \frac{f_{LX}^X}{f_{KX}^X} = \frac{f_{LX}^Y}{f_{KX}^X} \]
\[ \text{(5)} \]

which represents the slope of the transformation curve between \( X \) and \( Y \). It is interesting to note that the marginal conditions are not altered by the choice of the assignment functions to different government levels, as one might expect.

To close the model we need some conditions on the demand side. Our analysis aims to focus on the effects on the productive sector of tax evasion and on the relative cost of goods produced by the public sector, and for this reason it is more appropriate to fix the quantity of \( Y \) to be produced, i.e.

\[ Y = \bar{Y} \]
\[ \text{(6)} \]

Equations (1) (2) (5) and (6) define a general equilibrium for this simple economy.\(^{10}\) The perceived cost to produce \( Y \) depends on the organisation chosen for production. The total cost of production can in fact be written as:
\[ C_Y^C = (wSH_L^Y + rSH_K^Y)(1-t) \]
\[ C_Y^D = wSH_L^Y + rSH_K^Y \]
\[ \text{where } SH \text{ are the initial shares of factor inputs in the total cost of producing each output respectively.} \]

The perceived cost of production is higher in a decentralised system due to the separation between the production and the tax raising function. It is however important to note that this is a pure fiscal illusion effect since in both cases the actual cost, in terms of taxation, is the same.

### 3.2 Tax evasion

Let’s now suppose that a fraction of the income earned in the private sector can be evaded. Let’s define as \( L_D \) the number of hours the consumer declares to have worked for tax purposes and \( L_E \) the number of hours worked that evade taxation, so that the equality \( L_X = L_D + L_E \) has to hold.

\(^{10}\) Equation (1) is redundant for the definition of the equilibrium. \( X \) is in fact residual since it receives all the capital and labour that is not necessary to satisfy (2). Its use permits more straightforward results when tax
\( Y_E = w L_E \) is the income evaded.

There is a probability \( \pi(Y_E) \) \(^{11}\) of being caught; if caught, the consumers have to pay a fine proportional to the income evaded at rate \( s \). \(^{12}\)

In a full general equilibrium framework, consumers will choose \( L_X, L_D \) and all the other parameters simultaneously \(^{13}\) and the choice will be determined by the level of risk aversion of each individual and by the amount of public expenditure that Government devotes to tax audit. In this exercise we wish to concentrate on the impact of tax evasion on the relative cost of goods produced in the public sector and we do not consider the cost relating to auditing; for this reason we will assume that our representative consumer finds it optimal to evade a fraction \( w E \) of the income earned in the private sector. \(^{14}\) The effect on the production side consists of a change in the relative price of the two goods because tax evasion causes a reduction in the actual tax rate paid in the private sector. The actual tax rate \( t^* \) paid on the hours of work supplied to the private sector is in fact equal to:\(^ {15}\)

\[
t^* = \frac{wL^X - wL^*_E (1 - \pi(Y_E) - \pi(Y^*_E) s/t)}{wL^X} = \left[ 1 - \frac{L^*_E}{L^X} (1 - \pi(Y_E) - \frac{\pi(Y^*_E) s}{t}) \right] = t - \varepsilon
\]

where \( t \) is the tax scheduled.

Let’s now analyse the effect on the labour market. In a structure where the two producers pay the same gross wage \( w \) to workers, the net wage rate would be different because the labour employed in the production of \( X \) pays a higher expected net income than in the production of \( Y \). The consumer \(^{16}\) prefers to work for the private sector and the only way to make them stay in public production is to offer the same net salary rate \(^{17}\). This implies that the equilibrium wage evasion is introduced.

\(^{11}\) This assumption is standard in the literature on tax evasion. In this model it permits an interior solution also in the presence of agents that are risk-neutral.

\(^{12}\) According to American and Israeli laws, the marginal fine for evading taxes is imposed on the amount of evaded taxes (see Yitzhaki, 1974 & 1979). This alters the comparative static properties of the decision of how much to evade (Sandmo, 2005), but for the model at hand, given that tax evasion is not endogenous, the choice of the base for the fine is not relevant.

\(^{13}\) See, for example, Allingham and Sandmo (1972) and Cowell (1985).

\(^{14}\) See Cowell (1985).

\(^{15}\) If the consumer evades a part of his income, his expected tax bill on the work supplied in the private sector will be equal to: \( T_X = (1 - \pi(Y_E))(tw_X L_D) + \pi(Y_E)(tw_X L_X + sw_X L_E) \). Since \( L_x = L_D + L_E \), we can write the tax bill as:

\[
T_X = (1 - \pi(Y_E))(tw_X L_D) + \pi(Y_E)(tw_X L_X + sw_X L_E) = (tw_X L_D) - (tw_X L_E)(1 - \pi(Y_E) - \frac{\pi(Y_E) s}{t})
\]

\(^{16}\) This result can be generalised to a community of \( n \) individuals, provided they have the same degree of risk aversion.

\(^{17}\) The actual equilibrium salary might be different from \( w \), the wage rate offered when there is no evasion. For
rate has to satisfy the following condition:

\[ w^X (1 - t') = w^Y (1 - t) \]  

so that:

\[ w^Y = w^X \frac{(1 - t')}{(1 - t)} \]  

where \( w^X (1-t) \) represents the increase on the ongoing salary that has to be paid to workers in the public sector due to the differential in tax evasion.

Given the assumption of homogeneity in the production function, we can write the relative price for the public sector as:

\[ \frac{p_Y}{p_X} = \frac{rSH^X_Y + w^X (1-t + \varepsilon)SH^L_Y}{rSH^X_K + w^X SH^L_X} \]  

Starting from \( \varepsilon = 0 \), we can now study the effect of tax evasion on the system. First of all let’s examine the effect on the relative price for \( Y \). From (11) we can write:

\[ \frac{\partial p_Y}{\partial \varepsilon} = \frac{w^X}{(1-t)}SH^L_Y \frac{SH^L_X}{SH^X_K + w^X SH^L_X} \]  

which is certainly greater than zero. However, tax evasion has an important real effect on the economy. By altering the equilibrium conditions in production, it causes a loss to the economy in real terms. To show this let’s start by rewriting the new equilibrium condition that substitutes (3) and can be written as:

\[ \frac{f^X_{LX}}{f^X_{KX}} = \frac{f^Y_{LX}}{f^Y_{KX}} \frac{1-t + \varepsilon}{1-t} \]  

The effect on the production of \( Y \) and \( X \) can be shown in several ways; in this article we offer a graphical and an analytical solution. Figure two shows the change in the equilibrium due to a change in tax evasion.

The dimension of the Edgeworth box is defined by the total inputs that be used, i.e. \( K \) and \( L \). The isoquants for good \( Y \) are increasing NE; for good \( X \) the isoquants are increasing SW.

If tax evasion is not possible, the possible equilibria for this economy are represented by the line \( CC \) where the isoquants for good \( X \) are tangent to the corresponding isoquant for good \( Y \). In these point in fact the marginal rate of substitution for the two good is the same and it is also...
equal to the relative price of the two inputs.

The starting equilibrium is represented by a point which satisfies \( Y = Y^0 \) and determines the production of \( X = X^0 \) and the relative price of the two inputs \( AA \). Let’s now suppose that tax evasion is possible so that the actual tax rate for labour employed in \( X \) becomes \( t^* \). The relative input price for both production is no longer the same as shown in figure two. Since the cost for labour in \( Y \) has increased, capital will substitute labour and production should be contracted (Harberger, 1962). However, due to the restriction \( Y = Y^0 \), the new equilibrium will be in \( Ec \) where \( X' < X^0 \). In this simple economy where consumers have the same preferences tax evasion reduces the quantity of private production, hence welfare. Tax evasion is pure fiscal illusion, but it produces perverse effects on the real economy.

\[
\frac{\partial X}{\partial \varepsilon} = \varepsilon f_{K_x}^X \frac{f_{L_x}^X}{f_{K_x}^X} + \frac{f_{L_x}^Y}{f_{K_x}^X} + \frac{f_{L_x}^Y}{f_{K_x}^X} < 0
\]

which is negative and shows that the economy as a whole is worse off. The relative price for \( Y \) has increased and the total production of \( X \) has decreased. Equation (13) shows that the effect is directly related to the level of tax evasion, but it is independent from the level of tax evasion.
The analysis presented here does not allow us to study the effects on the demand for goods produced in the public sector. In general, we might predict that the rise in public expenditure will be lower in demand-driven models where the median voter might substitute public expenditure with private consumption due to the relative change in the price. In our analysis we have focused on the effect of tax evasion on the relative cost of public expenditure; in this context the result is unambiguous: there is a clear pushing-up effect on its relative cost which adds a further source of deadweight loss from those that the literature has studied so far.

4. Conclusions

In this article it is shown that an unbalanced level of tax evasion in different sectors of the economy due to institutional reasons might be the cause for an increase in the relative cost of providing services by the sectors where tax evasion is more difficult. The article examines a very simple, two-sector economy where income tax evasion in the private sector is not matched, as it is reasonable to assume, in the public sector. The mechanism is independent from the level of decentralisation with which production is organised and from the way such production is financed. From a welfare point of view, this paper shows that tax evasion, even in a very simple model where the whole effects of tax evasion are not considered, is not simply fiscal illusion. In the model presented here, in fact, we do not have considered the audit cost necessary to curb tax evasion and the deadweight losses arising from changes in consumer’s choice.

The result presented in this paper offers several interesting policy applications as regards finance instruments, the effects of tax evasion and the trade-off between tax evasion, public expenditure and the growth of the economy that can be studied using theoretical and/or applied general equilibrium models.

This model offers another interesting dimension to the debate on the optimal level of tax evasion and its policy implication. The literature usually considers the effects of tax evasion on distribution of the fiscal burden and consequences on the tax revenue; this paper shows that there is a cost increase in the production of goods and services that should also be considered.

The model presented here is focused on public expenditure but the same analysis can be extended to any case where two sectors have different rates of tax evasion for institutional reasons, i.e. the probability of being caught is different. In this case tax evasion allows some sectors to be more competitive since it reduces their labour costs.

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18 The analysis is presented in Levaggi (2000).
References


Appendix one

The general equilibrium effect of tax evasion can be studied solving the following equation system:

\[
\begin{bmatrix}
1 & -f_{LX}^X & -f_{LX}^X \\
0 & f_{LX}^Y + f_{[\bar{L}-L,X]}^Y & f_{LX}^Y + f_{[\bar{K}-K,X]}^Y \\
0 & \frac{f_{KX,KX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t} & \frac{f_{LX}^X}{f_{[\bar{K}-K,X]}^Y} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t}
\end{bmatrix}
\begin{bmatrix}
dX \\
dL_X \\
dK_X 
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

The RHS determinant can be written as:

\[
-f_{[\bar{L}-L,X]}^Y \left[ \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t} \right] - f_{[\bar{K}-K,X]}^X \left[ \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t} \right] > 0
\]

Substituting the first column with the first of the lhs determinant we get:

\[
\begin{bmatrix}
0 & -f_{LX}^X & -f_{LX}^X \\
0 & f_{LX}^Y + f_{[\bar{L}-L,X]}^Y & f_{LX}^Y + f_{[\bar{K}-K,X]}^Y \\
+ \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1}{1-t} & + \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t} & + \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t}
\end{bmatrix}
\begin{bmatrix}
dX \\
dL_X \\
dK_X 
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

which can be written as:

\[
\begin{bmatrix}
0 & -f_{LX}^X & -f_{LX}^X \\
0 & f_{LX}^Y + f_{[\bar{L}-L,X]}^Y & f_{LX}^Y + f_{[\bar{K}-K,X]}^Y \\
+ \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1}{1-t} & + \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t} & + \frac{f_{LX}^X}{f_{KX}^X} + \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1-t+\varepsilon}{1-t}
\end{bmatrix}
\begin{bmatrix}
dX \\
dL_X \\
dK_X 
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

The determinant can be written as:

\[
f_{KX}^X f_{[\bar{K}-K,X]}^Y \frac{f_{[\bar{L}-L,X]}^Y}{f_{[\bar{K}-K,X]}^Y} \frac{1}{1-t} \left\{ -f_{LX}^X + f_{[\bar{L}-L,X]}^Y \right\} < 0
\]
\[ \frac{\partial X}{\partial \varepsilon} = \frac{d\varepsilon}{dX} = \frac{f^X_{XY} \cdot f^Y_{(Y-K_X)X} \cdot f^Y_{(Y-L_X)X} \cdot \left( 1 - t \right)}{f^X_{YX} \cdot \left( 1 - t + \varepsilon \right)} \left( \frac{f^X_{L_X} + f^Y_{(Y-L_X)}}{f^Y_{(Y-L_X)} + f^Y_{(Y-K_X)X}} \right) \]

Given that:
\[ f^X_{L_X} = f^Y_{(Y-L_X)} \left( 1 - t + \varepsilon \right) \]
\[ f^X_{YX} = f^Y_{(Y-K_X)X} \left( 1 - t \right) \]

the effect is negative.

By re-arranging the numerator and the denominator we can write:
\[ \frac{\partial X}{\partial \varepsilon} = \frac{f^X_{L_X}}{f^X_{YX} + f^Y_{(Y-L_X)} + f^Y_{(Y-K_X)X} + f^Y_{(Y-L_X)} \left( 1 - t + \varepsilon \right)} < 0 \]

Which is equivalent to the expression in the text.
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0306 – Paolo M. PANTEGHINI, Guttorm SCHJELDERUP “Competing for Foreign Direct Investments: A Real Options Approach” (marzo)
0307 – Michele MORETTO, Gianpaolo ROSSINI “The Cost of Stock Options as an Incentive Device” (aprile)
0308 – Giulio PALERMO “The Ontology of Economic Power in a Critical Realist Perspective: The Orthodox Approach and Marx” (aprile)
0309 – Mario CASSETTI “A note on the long-run behaviour of Kaleckian models” (giugno)
0310 – Daniel ANKARLOO, Giulio PALERMO “Anti-Williamson a Marxian critique of New Institutional Economics” (giugno)
0311 – Sandye GLORIA-PALERMO, Giulio PALERMO “To What Extent is the Austrian Theory of Capital Austrian? Bohm-Bawerk and Marx Reconsidered” (giugno)
0312 – Paolo M. PANTEGHINI “A Dynamic Measure of the Effective Tax Rate” (ottobre)
0313 – John GEWEKE, Gianni AMISANO “Compound Markov Mixture Models with Applications in Finance” (ottobre)
0314 – Gianni AMISANO, Alessandra DEL BOCA “Profit Related Pay in Italy” (ottobre)
0315 – Michele POLO, Carlo SCARPA “Entry Without Competition” (ottobre)

Anno 2004
0401 – Francesco MENONCIN, Marco TRONZANO “Optimal Real Exchange Rate Targeting: A Stochastic Analysis” (gennaio)
0402 – Chiara D’ALPAOS, Michele MORETTO “La valutazione della flessibilità nel servizio idrico integrato” (maggio)
0403 – Francesco MENONCIN “Risk management for pension funds” (giugno)
0404 – Francesco MENONCIN “Risk management fora n internationally diversified portfolio” (agosto)
0405 – Franco SPINELLI, Carmine TRECROCI “Le determinanti del tasso di sconto in Italia negli anni 1876-1913: un’analisi empirica e documentale” (settembre)
0406 – Cesare DOSI “Enviromental Innovation, Wat of Attrition and Investment Grants” (settembre)
0407 – Chiara DALLE NOGARE, Matilde VASSALLI “Pressare on monetary policy: the Italian evidence” (ottobre)
0408 – Gianni AMISANO, Maria Letizia GIORGETTI “The Dynamics of Firms’ Entry and Diversification: A Bayesian Panel Probit Approach. A cross-country analysis” (ottobre)
0409 – Michele MORETTO, Gianpaolo ROSSINI “Start-up entry strategies: Employer vs. Nonemployer firms” (novembre)
0410 – Michele MORETTO, Paola VALBONESI “Dynamic Firm Regulation with Endogenous Profit-Sharing” (novembre)
0411 – Chiara DALLE NOGARE, Roberto RICCIUTI “Chief Executives’ Term Limits and Fiscal Policy Choices: International Evidence” (dicembre)

Anno 2005
0502 – Chiara D’ALPAOS, Cesare DOSI, Michele MORETTO “Concession Length and Investment Timing Flexibility” (gennaio)
0504 – Gianni AMISANO, Raffaella GIACOMINI “Comparing Density Forecasts via Weighted Likelihood Ratio Tests” (febbraio)
0505 – Roberto CASARIN “Stochastic Processes in Credit Risk Modelling” (marzo)
0506 – Paolo M. PANTEGHINI “S-Based Taxation under Default Risk” (luglio)
0507 – Michele MORETTO, Sergio VERGALLI “Migration Dynamics” (agosto)
0508 – Chiara D’ALPAOS, Paola VALBONESI “Una valutazione delle ipotesi di revisione del metodo tariffario normalizzato per il servizio idrico integrato” (novembre)
0509 – Raffaele MINIACI, Maria Laura PARISI “Which Plans to Reduce the Digital Divide? Policy Evaluation and Social Interaction” (ottobre)

Anno 2006
0601 – Francesco MENONCIN “The role of longevity bonds in optimal portfolios” (gennaio)
0603 – Roberto CASARIN, Carmine TRECROCI “Business Cycle and Stock Market Volatility: A Particle Filter Approach” (febbraio)
0604 – Chiara DALLE NOGARE, Matilde VASSALLI “A Pressure-Augmented Taylor Rule for Italy” (marzo)
0605 – Alessandro BUCCIOL, Raffaele MINIACI “Optimal Asset Allocation Based on Utility Maximization in the Presence of Market Frictions” (marzo)
0606 – Paolo M. PANTEGHINI “The Capital Structure of Multinational Companies under Tax Competition” (marzo)
0607 – Enrico MINELLI, Salvatore MODICA “Credit Market Failures and Policy” (gennaio)
0609 – Françoise FORGES, Enrico MINELLI “Affriat’s Theorem for General Budget Sets” (marzo)
0610 – Aviad HEIFETZ, Enrico MINELLI “Aspiration Traps” (marzo)
0611 – Michele MORETTO, Paolo M. PANTEGHINI, Carlo SCARPA “Profit Sharing and Investment by Regulated Utilities: a Welfare Analysis” (aprile)
0612 – Giulio PALERMO “Il potere come relazione sociale. Il caso dell’università baronale italiana” (giugno)
0613 – Sergio VERGALLI “Dynamics in Immigration Community” (luglio)
0614 – Franco SPINELLI, Carmine TRECROCI “Maastricht: New and Old Rules” (luglio)
0615 – Giulio PALERMO “La valutazione dei titoli scientifici dei docenti del Dipartimento di Scienze Economiche dell’Università di Brescia” (settembre)