A NOTE ON OPTIMAL TAX EVASION IN
THE PRESENCE OF MERIT GOODS

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A note on optimal tax evasion in the presence of merit goods

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Abstract
In a recent article Davidson, Lawrence and Wilson propose a model showing that, in the presence of distortionary taxation and goods of different quality, tax evasion can be an optimal device. Here, we show that this result, although quite interesting, cannot be generalised to a framework where Government activity consists of supplying merit goods and levying taxes to finance their provision.

JEL Classification: H26,H42
Key Words: merit goods, tax evasion.

1 Introduction
In a recent article Davidson, Martin and Wilson (2005), hereafter DMW, present a model showing that, in the presence of distortionary taxation, tax evasion can be an optimal strategy which, in some special cases, allows replication of a first best tax system. The model stands on four basic assumptions: a) there are two types of goods in the economy: a numeraire and another good that is produced in several qualities; b) goods are produced using Leontief technologies; c) consumers’ utilities are additive, and they differ in their evaluation of quality (the marginal utility of quality is however decreasing); d) Central Government levies taxes on the good with several varieties to finance the production of a preset level of public goods.

Although utility is linear, the assumption of decreasing marginal utility, combined with a non-linear aggregation of linear preferences, allows the authors to show that uniform tax is distortionary and that in some cases evasion can improve welfare. Given a probability of being caught, Central Government will then set a fine to a level that allows an optimal tax evasion; in this way the tax rate and the fine levied are not uniform and the authors say that welfare improves. In this work we use the framework proposed by DMW to study...
the scope for tax evasion in an environment where the taxed good is a merit good. We assume that the Government produces the low quality good and subsidizes its consumption whereas the private sector produces the high quality good (which is not subsidized). We believe that this environment is quite suitable to represent Government activity which is more oriented towards the production of merit rather than public goods.

2 Merit goods, public provision and optimal tax

As in DMW, we assume a simple economy where two classes of goods are produced: a quality homogenous numeraire, and a private good whose quality can be either low \((\theta_L)\) or high \((\theta_H > \theta_L)\). Each agent is endowed with a given amount of the numeraire and he decides whether to buy a single unit of the other good and of which quality. Departing from DMW, we assume that the two-quality product is a merit good. Accordingly, the Government produces and promotes the consumption of the merit good by subsidizing the low quality variety \((\theta_L)\) at rate \((1 - \rho)\) so that the price paid by the final user is a fraction \(\rho\) of the low quality good price \(P_L\). Only the low quality is subsidized in order to reduce as far as possible the burden of Government activity. We believe that health care and education could be two relevant examples. In fact, they are produced both in the public and the private sector, but with different standards as regards quality\(^3\) and prices. The utility function of agent \(i\) who buys the good whose quality is \(j \in \{L, H\}\) can be written as

\[
U_i = E_i - \rho_j P_j(\theta_j) + \alpha_i v(\theta_j),
\]

where \(v\) is an increasing and concave function of the quality \((\theta_j)\) and is the same for any agent, \(E_i\) is the numeraire endowment of agent \(i\) (which can be transformed into either labour or capital), \(P_j(\theta_j)\) is the price of the good \(j\) as a strictly increasing function of the quality \(\theta_j\), and \(\alpha_i \in [0, 1]\) is a preference agent specific parameter and is distributed according to the continuous density function \(h(\alpha)\). Although \(h(\alpha)\) is known by the Government, each single value of \(\alpha\) is private information to the consumer. This is the reason why subsidies and taxation can be applied to goods, but they cannot depend on personal characteristics of the consumers. This means that, as in DWM, an optimal tax system cannot be defined. The subsidy \(\rho_j\) is either zero for the high quality good or constant for the low quality good (i.e. \(\rho_L = \rho\) and \(\rho_H = 1\)). The reason for subsidizing only the low quality good is twofold: (i) since the public expenditure is always distortionary, it should be kept to the minimum, and (ii)

\(^2\)In OECD countries about one fourth of public expenditure is represented by public goods and their share is decreasing through time.

\(^3\)The difference in quality levels might be related: (i) to the number of hotel services provided in private hospitals as regards health care, and (ii) to the average number of pupils in each classroom as regards education.
if the low quality merit good satisfies consumer needs, there is no reason to subsidize another good that only differs in its quality content.

Given the absence of depreciation and assuming zero interest rate, zero profits and a Leontief production function, both inputs are paid the same cost $W$ and in equilibrium prices for the goods must satisfy:

$$P_j = W_j (1 + t_j),$$

where $t_j$ is the tax on good $j$. Any agent $i$ may behave in three different ways as regards the choice of the merit good:

1. he does not buy it; this happens if and only if he has a preference $\alpha_i$ such that
   $$E_i - \rho P_L (\theta_L) + \alpha_i v (\theta_L) < E_i \Leftrightarrow \alpha_i < \frac{\rho P_L (\theta_L)}{v (\theta_L)} \equiv \alpha_L,$$

2. he buys the high quality product; this happens if and only if he has a preference $\alpha_i$ such that
   $$E_i - P_H (\theta_H) + \alpha_i v (\theta_H) > E_i - \rho P_L (\theta_L) + \alpha_i v (\theta_L)$$
   $$\Leftrightarrow \alpha_i > \frac{P_H (\theta_H) - \rho P_L (\theta_L)}{v (\theta_H) - v (\theta_L)} \equiv \alpha_H,$$

3. he buys a low quality product, this happens if and only if he has a preference $\alpha_i$ such that $\alpha_L < \alpha_i < \alpha_H$.

Other things being equal, the level of $\rho$ discriminates between the market for the public and the private varieties since it lowers $\alpha_L$ and increases $\alpha_H$. However, in the context of a full general equilibrium, this conclusion might be too simplistic since the effect of the tax system should also be studied. To do so, let’s start by studying a framework without tax evasion and where the two goods can be taxed at different rates. Then, the Government solves the following problem:

$$\max_{t_H, t_L} \int_{\alpha_L}^{\alpha_H} (\alpha v_L - \rho P_L) h (\alpha) d\alpha + \int_{\alpha_H}^{t_H} (\alpha v_H - P_H) h (\alpha) d\alpha$$

subject to the budget constraint

$$(1 - \rho) \int_{\alpha_L}^{\alpha_H} P_L h (\alpha) d\alpha = \int_{\alpha_H}^{1} t_H W_H h (\alpha) d\alpha + \int_{\alpha_L}^{\alpha_H} t_L W_L h (\alpha) d\alpha. \quad (2)$$

Before showing the main result, let’s introduce a definition.

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4DMW choose to model the tax rate in a slightly different way. Let us call $\tau_j$ their tax rate, then in their model the link between price $P_j$ and cost $W_j$ is given by $P_j = W_j / (1 - \tau_j)$. We made our choice in order to simplify the ongoing computations and, nevertheless, our result can be traced back to that of DMW through the following equalities:

$$\frac{\tau_j}{1 - \tau_j} = t_j \Leftrightarrow t_j = \frac{t_j}{1 + t_j}.$$ 

5We set $v (\theta_j) \equiv v_j$ and $P_j (\theta_j) \equiv P_j$. 

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3
Definition 1 A tax-subsidy system \((t_H^*, t_L^*, \rho^*)\) is said to be effective if and only if the indirect utility function of the agent is different from the one obtained with \((t_H^* = 0, t_L^* = 0, \rho^* = 1)\).

Let’s define

\[
H(X) \equiv \int_0^X h(\alpha) \, d\alpha,
\]

as the probability that the parameter \(\alpha\) of an agent takes values lower than \(X\). Then we can conclude what follows.

Proposition 2 Given Problem (1) under the constraint (2) and if \(W_H (1 + t_H^*) < v_H - v_L\), then the only effective tax-subsidy system is given by

\[
\begin{align*}
\rho^* &= 0, \\
t_L^* &= -1, \\
t_H^* &= \frac{W_L}{W_H} \frac{H \left( \frac{W_H (1 + t_H^*)}{v_H - v_L} \right)}{1 - H \left( \frac{W_H (1 + t_H^*)}{v_H - v_L} \right)}.
\end{align*}
\]

Proof. See Appendix 3. □

Accordingly, we have \(\alpha_L^* = 0\) which means that the optimal taxes and subsidy are set so that everybody will buy one of the goods. Central Government optimally finances the whole consumption of the merit good. This result is quite interesting because in this context tax evasion in its classical meaning is simply not possible. The burden of providing the merit good to the consumers that ask for the low quality good cannot be shifted. The only effect tax evasion might have is to reduce the number of users of the lower quality service by making it more convenient for the marginal consumer to shift to the higher quality consumption. In this case, public expenditure and the tax burden would shrink. To check whether this effect exists and if it is welfare improving, let’s first examine the decision of the firm to evade.

As in DMW, we assume that the final consumer benefits from tax evasion through a reduction in the price of the good he buys. Tax evasion is possible, but there is a probability \(\pi\) of being caught. In this case, the firm should pay a fine \(f\) proportional to the cost of production. Firms are assumed to be risk neutral; this means that they evade if

\[
\pi f W_H < t_H W_H,
\]

which can be written as

\[
f < \frac{t_H}{\pi}.
\]

Accordingly, in order to avoid tax evasion, Central Government should set \(f\) such that \(f \geq \frac{t_H}{\pi}\). In DMW the implicit assumption is that Central Government chooses to set \(f\) to a level that allows tax evasion. In this way the effective
tax rate paid in the two sectors is different and welfare improves. Does this result hold in the presence of a merit good? Our answer is no. To show this, let’s assume, as in DMW, that Central Government foresees that the firms in the private sector may evade if the fine and the probability of being caught are suitably chosen. If such an instrument were welfare improving, we should expect the Government to allow tax evasion. We introduce this assumption in our model and determine Central Government optimal policy. When evasion is taken into account, the price of the high quality good could be either $W_H$ when the firm evades, or $W_H(1 + f)$ when the firm evades and is caught. Accordingly, the expected price for good with quality $H$ will be

$$P_H = (1 - \pi)W_H + \pi \sigma W_H = W_H(1 + \pi f),$$

and the budget constraint for the Government becomes

$$(1 - \rho) \int_{\alpha_L}^{\alpha_H} P_L h(\alpha) \, d\alpha = \int_{\alpha_L}^{1} \pi \sigma W_H h(\alpha) \, d\alpha + \int_{\alpha_L}^{\alpha_H} t_L W_L h(\alpha) \, d\alpha,$$

while the objective function is

$$\max_{t^*_L, \sigma} \int_{\alpha_L}^{\alpha_H} (\alpha v_L - \rho P_L) h(\alpha) \, d\alpha + \int_{\alpha_H}^{1} (\alpha v_H - P_H) h(\alpha) \, d\alpha.$$

We can immediately see that the new problem and constraint have the same structure as Problem (1) and constraint (2) once $f$ is substituted by $t_H$. Accordingly, the solution is

$$\rho^* = 0,$$

$$t^*_L = -1,$$

$$f^* = \frac{1}{\pi} \frac{H_H}{W_H 1 - H_H} = \frac{t_H^*}{\pi}.$$

Thus, Central Government sets the fine at the lowest level not allowing tax evasion. This means that in the presence of a merit good it is not optimal to artificially decrease the price of the good produced in the private sector through tax evasion. In this system tax evasion might of course still exist if, as it is plausible to assume, Central Goverment is not able to observe the technology of production and the exact cost of production.\(^6\) If this is the case, there will be tax evasion in equilibrium, but at the cost of decreasing total welfare. This means that even in this very simple model where the costs of tax evasion in terms of controls and marginal cost of public funds (Levaggi, 2007) are not considered, tax evasion is not welfare improving. Thus, tax evasion is not always an optimal tax device, something that has been pointed out also by Davidson, Martín and Wilson (2006) themselves. We belive that the article proposed by the authors is a good starting point for studying the problem from a different perspective, in ways that the literature has not explored so far.

\(^6\)For example, if the fine is levied, as in DMW, on the total assets of the firm, the value of capital might not be observed with precision.
3 Demonstration of Proposition 2

Given Problem (1) and the constraint (2), we can write the Lagrange by using the function $H$ defined in (3) as follows

$$
\mathcal{L} = v_L \int_{\alpha_L}^{\alpha_H} \alpha h(\alpha) d\alpha - \rho W_L (1 + t_L) (H_H - H_L) + v_H \int_{\alpha_H}^{\alpha_L} \alpha h(\alpha) d\alpha - W_H (1 + t_H) (1 - H_H) + \lambda ((1 - \rho (1 + t_L)) W_L (H_H - H_L) - t_H W_H (1 - H_H)),
$$

where: $\lambda$ is the Lagrange multiplier, $H_H \equiv H(\alpha_H)$, $H_L \equiv H(\alpha_L)$, $h_H \equiv h(\alpha_H)$, and $h_L \equiv h(\alpha_L)$. We recall that

$$
\frac{\partial \alpha_H}{\partial t_L} = -\frac{\rho W_L}{v_H - v_L}, \quad \frac{\partial \alpha_L}{\partial t_L} = \frac{\rho W_L}{v_L},
$$

$$
\frac{\partial H_H}{\partial t_L} = -h_H \frac{\rho W_L}{v_H - v_L}, \quad \frac{\partial H_L}{\partial t_L} = h_L \frac{\rho W_L}{v_L - v_L},
$$

and accordingly, the system of the first derivatives of $\mathcal{L}$ with respect to $t_H$, $t_L$, $\rho$, and $\lambda$ is

$$
\frac{\partial \mathcal{L}}{\partial t_H} = \lambda \frac{(1 - \rho (1 + t_L)) W_L + t_H W_H}{v_H - v_L} - (1 + \lambda) \frac{1 - H_H}{h_H} = 0, \quad (4)
$$

$$
\frac{\partial \mathcal{L}}{\partial t_L} = -\rho \left[ \lambda \frac{(1 - \rho (1 + t_L)) W_L \left( \frac{h_H}{v_H - v_L} + \frac{h_L}{v_L} \right)}{v_H - v_L} + \lambda \frac{t_H W_H h_H}{v_H - v_L} + (1 + \lambda) (H_H - H_L) \right] = 0, \quad (5)
$$

$$
\frac{\partial \mathcal{L}}{\partial \rho} = -(1 + t_L) \left[ \lambda \frac{(1 - \rho (1 + t_L)) W_L \left( \frac{h_H}{v_H - v_L} + \frac{h_L}{v_L} \right)}{v_H - v_L} + \lambda \frac{t_H W_H h_H}{v_H - v_L} + (1 + \lambda) (H_H - H_L) \right] = 0, \quad (6)
$$

$$
\frac{\partial \mathcal{L}}{\partial \lambda} = (1 - \rho (1 + t_L)) W_L (H_H - H_L) - t_H W_H (1 - H_H) = 0. \quad (7)
$$

We highlight that Equations (5) and (6) have a factor in common. Accordingly, we can distinguish three main cases.

1. $\rho^* \neq 0$ and $t^*_L \neq -1$: in this case Equation (5) can be divided by $\rho$ and Equation (6) can be divided by $(1 + t_L)$. So, the system of the first order conditions contains just three equations:

$$
0 = \lambda \frac{(1 - \rho (1 + t_L)) W_L + t_H W_H}{v_H - v_L} - (1 + \lambda) \frac{1 - H_H}{h_H},
$$

$$
0 = \lambda \frac{(1 - \rho (1 + t_L)) W_L \left( \frac{h_H}{v_H - v_L} + \frac{h_L}{v_L} \right) + \lambda \frac{t_H W_H h_H}{v_H - v_L}}{v_H - v_L} + (1 + \lambda) (H_H - H_L),
$$

$$
0 = (1 - \rho (1 + t_L)) W_L (H_H - H_L) - t_H W_H (1 - H_H).
$$
If we take the value \((1 - \rho (1 + t_L)) W_L\) from the last equation and we substitute it in the first two equations we have

\[
\begin{align*}
t_H^* &= 0, \\
\lambda^* &= -1,
\end{align*}
\]

and, accordingly

\[1 - \rho (1 + t_L) = 0,\]

which means that

\[t_L^* = -1 + \frac{1}{\rho},\]

for any value of \(\rho\). Since in this case

\[
\alpha_H^* = \frac{W_H - W_L}{v_H - v_L}, \quad \alpha_L^* = \frac{W_L}{v_L},
\]

\[
P_H^* = W_H, \quad P_L^* = \frac{W_L}{\rho},
\]

then the indirect utility function is

\[
v_L \int_{\alpha_L}^{\alpha_H} \alpha h (\alpha) d\alpha - W_L (H_H^* - H_L^*) + v_H \int_{\alpha_H}^{1} \alpha h (\alpha) d\alpha - W_H (1 - H_H^*),
\]

which is exactly the same as we would have without any public intervention. Given Definition 1, then this tax-subsidy system is not effective.

2. Either \(\rho^* = 0\) and \(t_L^* \neq -1\) or \(\rho^* \neq 0\) and \(t_L^* = -1\): in this case either Equation (5) or Equation (6) is always verified. Furthermore, we have \(\alpha_H^* = 0\) and \(H_L^* = 0\). In both cases the system of the first order conditions contains the same three equations:

\[
\begin{align*}
\lambda W_L + t_H W_H &- (1 + \lambda) \frac{1 - H_H}{h_H} = 0, \\
\lambda W_L \left( \frac{h_H}{v_H - v_L} + \frac{h_L}{v_L} \right) + \lambda t_H W_H h_H &- (1 + \lambda) H_H = 0, \\
W_L H_H - t_H W_H (1 - H_H) & = 0,
\end{align*}
\]

which has no solution for \(t_H, t_L,\) and \(\rho\). Accordingly, this case is not relevant.

3. \(\rho^* = 0\) and \(t_L^* = -1\). In this case \(\alpha_L^* = 0\) and \(H_L^* = 0\) and Equations (5) and (6) are always verified for any values of \(t_H\) and \(\lambda\). Thus, we still have to solve the following system of two equations

\[
\begin{align*}
\lambda W_L + t_H W_H &- (1 + \lambda) \frac{1 - H_H}{h_H} = 0, \\
W_L H_H - t_H W_H (1 - H_H) & = 0,
\end{align*}
\]
from which we easily obtain

\[
    \lambda^* = \left( \frac{W_L}{v_H - v_L} \frac{h_H}{(1 - H_H)^2} - 1 \right)^{-1},
\]

\[
    t^*_H = \frac{W_L}{W_H} \frac{H_H}{1 - H_H}.
\]

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