OPTIMAL INVESTMENT AND FINANCIAL STRATEGIES UNDER TAX RATE UNCERTAINTY

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Optimal Investment and Financial Strategies 
under Tax Rate Uncertainty*

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Abstract

In this paper we apply a real-option model to study how tax rate uncertainty affects a firm’s decisions about both the timing and the source of finance of an investment project. We show that debt finance (i) encourages entry and (ii) mitigates the effect of tax rate uncertainty on entry timing.

JEL Classification: H2

Keywords: capital levy, corporate taxation, default risk, real options.

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1 Introduction

The empirical evidence shows that tax rate changes are frequent all over the world. In most cases, taxpayers cannot foresee future tax changes. This means that tax changes are a source of uncertainty. Tax policy uncertainty arises either when a government announces a tax rate change which will not be implemented after or when an unexpected tax change takes place. When therefore a government announces a tax rate change which will not be implemented after or when an unexpected tax change takes place, a commitment failure takes place.

As pointed out by Mintz (1995, p. 61): "When capital is sunk, governments may have the irresistible urge to tax such a capital at a high rate in the future. This endogeneity of government decisions results in a problem of time consistency in tax policy, whereby governments may wish to take actions in the future that would be different from what would be originally planned". In this case, the commitment failure leads to the well-known "capital levy problem",\(^1\) which is related to a firm's fear that a government can decide to raise taxes on capital already invested. Indeed, firms are aware that the government can undertake actions different from those initially planned and try to anticipate its tax choices.\(^2\) For this reason the capital levy problem deserves particular attention in an international setting.

It is worth noting that, over the last two decades, many countries cut tax rates in order attract foreign investment. Due to tax competition, therefore, firms have been operating in a tax-rate-cut scenario where further reductions may occur in the future. Whatever the sign of the tax rate change is, however, tax rate uncertainty is a fairly important problem and must be analyzed with appropriate techniques. Since the beginning of 2000s, the real-option literature has studied this phenomenon. This approach well fits with the tax rate uncertainty problem. As explained by Pindyck (2004, p. 12): "sunk costs do matter in decision-making when those costs have yet to be sunk".

\(^1\)Eichengreen (1990) points out that if the delay between a tax proposal and its implementation is substantial, capital mobility could make this additional tax burden ineffective.

\(^2\)For instance, Marceau and Smart (2003) focus on capital already invested. They assume that firms choose the initial level of capital and, after the government has announced its tax policy, can adjust their capital accumulation. Given the existence of investment adjustment costs, therefore, capital is assumed to be partially irreversible.
This means that the effects of tax rate uncertainty on firms’ decisions must be analyzed from an ex ante perspective, i.e., when firms are still free to choose not only whether but also when to invest. Böhm and Funke (2000) show that the effects of tax policy uncertainty may be small. Agliardi (2001) shows that uncertain investment tax credits can delay investment. Panteghini (2001a, 2001b) uses a Poisson process for the tax rate and proves that investment may be unaffected by tax policy uncertainty, if an ACE-type system is applied. Niemann (2004) defines two neutrality conditions: first-order neutrality, that requires the complete ineffectiveness of taxation on investment decisions; second-order neutrality, which means that the stochastic nature of taxation does not alter investment decisions. Moreover, Niemann (2006) analyses combined tax rate and tax base uncertainty by assuming a stochastic tax payment. He shows that uncertainty of tax payments has an ambiguous impact on investment timing.\(^3\)

It is worth noting that all these articles deal with an equity-financed investment. As we know, however, investment and financial decisions are related and, for this reason, must be studied together. To provide a more realistic analysis, we will therefore focus on a representative firm that can decide not only whether and when to invest but also how to finance its project. Using a standard trade-off approach we will study two different scenarios: a standard capital-levy one, where the tax rate is expected to rise, and a tax-competition one, where there is a downward trend in tax rates. We will then analyze the effects of tax rate uncertainty on both investment timing and capital structure. In particular, we will show that the higher the probability of a tax rate increase, the earlier a company will invest. In other words, a firm is induced to anticipate entry and take advantage of the lower current tax rate. As will be shown, the higher the probability of a tax rate increase, the higher is a firm’s leverage. More importantly, we will show that debt-finance can mitigate the effects of tax-rate uncertainty on investment decisions.

The structure of the article is as follows. Section 2 describes the model. Section 3 shows our main findings and discusses how the capital levy problem affects a firm’s financial decision. Section 4 summarizes our main findings and discusses its implications.

\(^3\)See also Sureth (2002).
2 The model

In this section we introduce an EBIT-based model in the spirit of Goldstein et al. (2001). By focusing on cash flows rather than stocks, we can better describe the investment and financial strategies of an infinitely-lived risk-neutral firm.\(^4\)

Let us denote \( \Pi_t \) as the firm’s Earning Before Interest and Taxes (EBIT). We assume that it evolves as follows:

\[
\frac{d\Pi_t}{\Pi_t} = \alpha dt + \sigma dz_t, \text{ with } \Pi_0 \geq 0,
\]

where \( \alpha \) is the expected rate of growth, \( \sigma \) is the instantaneous standard deviation of \( \frac{d\Pi_t}{\Pi_t} \) and \( dz_t \) is the increment of a Brownian motion. Moreover, we introduce the following hypotheses.

**Assumption 1** The firm must pay a sunk start-up cost \( I \) to undertake a risky project.

**Assumption 2** The firm can borrow from a perfectly competitive risk-neutral credit sector, characterized by a given risk-free interest rate \( r \).

**Assumption 3** The firm can decide how much to borrow by choosing a non-renegotiable coupon \( C \).

**Assumption 4** Debt is protected.

**Assumption 5** The cost of default is equal to \( vC \) with \( v > 0 \).\(^5\)

According to Assumption 1, the firm must pay a sunk cost. This leads to investment irreversibility. Assumption 2 describes a simple framework where lenders are price-taker and become shareholders in the event of default. In line with Leland (1994), the firm chooses an optimal coupon (Assumption

\(^4\)For a similar analysis with risk-averse firms, see Niemann and Sureth (2004, 2005).

\(^5\)The quality of results does not change if, like Leland (1994), we assume that default costs are proportional to the firm’s value.
for simplicity, the capital structure is static, i.e., financial policy cannot be reviewed later. Moreover, according to Assumption 4, debt is protected, i.e., default occurs when the firm’s profit, net of its debt obligations, is nil. When finally default takes place, a sunk default cost, equal to $vC$, is faced (Assumption 5).

Let us next introduce taxation. We denote $\tau$ as the tax rate and assume that interest payments are fully deductible. As regards the treatment of the lender’s receipts, it is well-known that effective tax rates on capital income are fairly low. With no loss of generality, therefore, we assume that the lender’s pre-default tax burden is nil, so that her after-tax profit function at any instant $t$ is simply $C$. When, however, the lender becomes shareholder, she is subject to corporate taxation.

The after-tax firm’s profit function at time $t$ is thus equal to

$$\Pi^N = (1 - \tau) (\Pi_t - C).$$

Given Assumption 4, therefore, default takes place when $\Pi^N = 0$. This means that the default threshold point is $C$. Hereafter, we will omit the time subscript $t$.

Let us finally model tax rate uncertainty. We assume that the tax rate follows a Poisson process. Given an initial tax rate $\tau_0$, at any short time interval $dt$ there is a probability $\lambda dt$ that the tax rate augments to $\tau_1$. Therefore we can write:

$$d\tau = \begin{cases} 0 & \text{w.p. } 1 - \lambda dt, \\ \Delta \tau & \text{w.p. } \lambda dt, \end{cases}$$

Given $C$ and the market conditions, the market value of debt can be calculated. It is worth noting that, in the absence of arbitrage, setting the coupon first, and then, calculating the market value of debt is equivalent to first choose the value of debt and then calculate the effective interest rate. The ratio between $C$ and the market value of debt is equal to the interest rate (which is given by the sum between $r$ and the default risk premium).

Of course, the absence of debt renegotiation is not realistic, although it does not affect the qualitative properties of the model. For a detailed analysis of dynamic trade-off strategies, with costly debt renegotiation, see e.g. Goldstein, Ju and Leland (2001), and Hennessy and Whited (2005).

For further details on default conditions see Brennan and Schwartz (1978), and Smith and Warner (1979). As pointed out by Leland (1994) minimum net-worth requirements, implied by protected debt, are common in short-term debt financing. For a comparison with default under unprotected debt financing see Panteghini (2007a).
where $\Delta \tau = \tau_1 - \tau_0$. Given (3), we can focus on two different scenarios:

1. a capital-levy scenario, where the tax rate is expected to rise (i.e., $\Delta \tau > 0$);

2. a tax-competition scenario, where the tax rate is expected to decrease (i.e., $\Delta \tau < 0$).

### 2.1 The value function

Given these assumptions we can now calculate the firm’s value function. We will follow a backward approach. So, we will first calculate the value function after the expected tax rate change: in this case the relevant rate is $\tau_1$ and the value function will be denoted as $V_1$. The we will focus on the before-tax-change scenario where the current statutory rate is $\tau_0$.

Let us start with the after-change scenario. The value function $V_1(\Pi; C)$ is given by the sum between the equity value $E_1(\Pi; C)$ and the debt value $D_1(\Pi; C)$ net of the investment cost $I$. As shown in Appendix A, the value function amounts to:

$$V_1(\Pi; C) = \left\{ \begin{array}{ll}
\frac{(1-\tau_1)\Pi}{\delta} - I, & \text{after default,} \\
\frac{(1-\tau_1)\Pi}{\delta} + \frac{\tau_1 C}{r} - \left( \frac{\tau_1}{r} + \nu \right) C \left( \frac{\Pi}{C} \right)^{\beta_2} - I, & \text{before default.}
\end{array} \right. \quad (4)$$

The term $\frac{(1-\tau_1)\Pi}{\delta}$ measures the gross value of an unlevered firm; $\frac{\tau_1 C}{r}$ is the tax benefit due to deductibility of the interest expenses. The third term $- \left( \frac{\tau_1}{r} + \nu \right) C \left( \frac{\Pi}{C} \right)^{\beta_2}$ measures the contingent value of the default cost. This means that, in the event of default, a firm faces a sunk cost $\nu C$ and loses the tax benefit of interest deductibility $\left( \frac{\tau_1 C}{r} \right)$. This cost is contingent on the event of default: $\left( \frac{\Pi}{C} \right)^{\beta_2}$, with $\beta_2 < 0$ (see Appendix A) is the contingent value of $1\mathcal{E}$ in the event of default.

Given these assumptions, we can calculate a firm’s value function before the tax rate change, which we denote with $V_0$. As shown in Appendix A, $V_0$ is given by the sum of the equity value $E_0$ and the debt value $D_0$ before the tax rate change, net of the investment cost $I$ and it can be written as follows:

$$V_0(\Pi; C) = E_0(\Pi; C) + D_0(\Pi; C) - I = V_1(\Pi; C) + X(\Pi; C) + Y(\Pi; C) - I, \quad (5)$$
where \( X(\Pi; C) \equiv E_0(\Pi; C) - E_1(\Pi; C) \) and \( Y(\Pi; C) \equiv D_0(\Pi; C) - D_1(\Pi; C) \) are the expected change in the equity and debt value, respectively, contingent on the tax rate increase. In symbols

\[
X(\Pi; C) = (\tau_1 - \tau_0) \left[ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \right]
\]

and

\[
Y(\Pi; C) = \frac{\tau_1 - \tau_0}{\delta + \lambda} C \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)},
\]

where \( \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \), with \( \beta_2(\lambda) < 0 \) (see again Appendix A) is the contingent value of \( 1e \) in the event of default, under tax rate uncertainty.

If we compare the tax certainty case with the tax uncertainty one, we can see that contingent evaluation changes: instead of \( \left( \frac{\Pi}{C} \right)^{\beta_2} \) we have \( \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \). Since \( \beta_2(\lambda) < \beta_2 < 0 \), the inequality \( \left( \frac{\Pi}{C} \right)^{\beta_2} > \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \) holds. This means that tax rate uncertainty affects the contingent evaluation of default. In particular, the contingent value of \( 1e \) is lower than under tax rate certainty.

As can be seen, both \( X(\Pi; C) \) and \( Y(\Pi; C) \) account for the tax rate change and are therefore proportional to the differential \( (\tau_1 - \tau_0) \). In particular, \( X(\Pi; C) \) measures the effects of the tax rate change on the value of equity. As such, it accounts for the value change both before default, i.e., with term

\[
(\tau_1 - \tau_0) \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right),
\]

but also for the contingent effect of default, which is given by the term

\[
- (\tau_1 - \tau_0) \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)}
\]

Function \( Y(\Pi; C) \), which measures the expected change in the debt value, is proportional to the coupon \( C \) and is contingent on the event of default (i.e., it depends on \( \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \)).

### 2.2 The option value

Let us next calculate a firm’s option to invest. To do so, we denote the option value as \( O_1 \) and \( O_0 \), under tax rate certainty and uncertainty, respectively. Again, we will follow a backward approach.
Let us start with the after-change scenario. As shown in Appendix B, the option value under tax rate certainty is equal to:

\[
O_1 (\Pi; C; \bar{\Pi}) = \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \left[ \frac{(1 - \tau_1) \bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left( \frac{\bar{\Pi}}{C} \right)^{\beta_2} - I \right].
\]

where \( \bar{\Pi} \) is the threshold level of EBIT, above which investment is profitable. As can be seen, the option value is given by the product between the term \( \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \), which measures the present value of \( 1\varepsilon \) contingent on the entry decision, and

\[
\frac{(1 - \tau_1) \bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left( \frac{\bar{\Pi}}{C} \right)^{\beta_2} - I,
\]

which is the Net Present Value at point \( \Pi = \bar{\Pi} \), i.e., when investment is made.

As shown in Appendix B, the option value under tax rate uncertainty \( O_0 \) can be written as follows:

\[
O_0 \left( \Pi; C; \hat{\Pi}; \bar{\Pi} \right) = O_1 \left( \Pi; C; \bar{\Pi} \right) + Z \left( \Pi; C; \bar{\Pi}; \hat{\Pi} \right),
\]

where \( \hat{\Pi} \) is the threshold level of EBIT, above which investment is profitable, and

\[
Z \left( \Pi; C; \hat{\Pi}; \bar{\Pi} \right) \equiv O_0 \left( \Pi; C; \hat{\Pi}; \bar{\Pi} \right) - O_1 \left( \Pi; C; \bar{\Pi} \right)
\]

is the expected change in the option value, contingent on the tax rate increase. It amounts to

\[
\left[ V_0 \left( \hat{\Pi}; C \right) - V_1 (\Pi; C) \left( \frac{\hat{\Pi}}{\Pi} \right)^{\beta_1} \right] \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1 (\lambda)},
\]

where \( V_0 \left( \hat{\Pi}; C \right) \) and \( V_1 (\Pi; C) \) are the value functions at the optimal threshold level \( \hat{\Pi} \) and \( \Pi \), respectively. Moreover, \( \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1 (\lambda)} \) is the contingent value of \( 1\varepsilon \) invested when \( \Pi \) reaches \( \hat{\Pi} \). Finally, \( \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \) measures the wedge caused by tax rate uncertainty on contingent evaluation. As can be seen, under tax rate uncertainty we have \( \hat{\Pi} < \bar{\Pi} \), and therefore, the wedge is \( \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} < 1.9 \)

---

9 If tax rate uncertainty vanishes, the equality \( \hat{\Pi} = \bar{\Pi} \) holds and \( \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \) goes to unity.
3  A numerical analysis of a firm’s problem

As we have shown, tax-rate uncertainty affects not only a firm’s Net Present Value but also its investment option. To analyze its overall effect let us next study the firm’s choices on the optimal coupon $C$ and the investment timing.

To do so we will proceed as follows: (i) we will first study the problem when tax rate is certain and equal to initial value $\tau_0$; (ii) then we will deal with the problem when tax-rate uncertainty matters.

(i) Under tax-rate-certainty the firm’s problem is:

$$\max_{\Pi > 0, C > 0} \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} \left[ \frac{(1 - \tau_0) \hat{\Pi}}{\delta} + \frac{\tau_0 C}{r} - (\tau_0 + rv) \frac{C}{r} \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_2} - I \right], \tag{8}$$

where the above objective function is obtained by substituting $\tau_0$ to $\tau_1$ in (6). As shown in Appendix C, solutions to (8) are:

$$\begin{align*}
\hat{\Pi}^* &= \frac{1}{1 + m \beta_1 \beta_2} \frac{rL}{1 - \tau_0}, \\
C &= \left( \frac{1}{1 - \beta_2 (1 - \tau_0 + rv)} \right)^{\frac{1}{\beta_2}} \hat{\Pi}^*,
\end{align*}$$

where $m \equiv \frac{\tau_0}{1 - \tau_0} \beta_2 - 1 \left( \frac{1}{1 - \beta_2 (1 - \tau_0 + rv)} \right)^{\frac{1}{\beta_2}} > 0$ measures the effect of leverage on the investment timing. It is also easy to show that both $\hat{\Pi}^*$ and $C$ increase with $\tau_0$. This means that a higher tax rate delays entry and encourages debt finance.

(ii) Under tax-rate-uncertainty the firm’s problem is as follows:

$$\max_{\Pi > 0, C > 0} O_0 \left( \Pi; C; \hat{\Pi}; \hat{\Pi} \right), \tag{9}$$

where

$$O_0 \left( \Pi; C; \hat{\Pi}; \hat{\Pi} \right) = V_1(\hat{\Pi}; C) \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} + V_0 \left( \hat{\Pi}; C \right) - V_1(\hat{\Pi}; C) \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda)}.$$ 

Solving (9) gives the optimal values $C^*$ and $\hat{\Pi}^*$. However, whereas the tax-rate-certainty problem has a closed-form solution, problem (9) has not. For this reason, we must analyze a firm’s strategy by applying a numerical approach. We will numerically study the solution of problem (9) by using the benchmark parameter values of Table 1.
Table 1. The parameter values for numerical solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$\sigma$</td>
<td>.4</td>
</tr>
<tr>
<td>$I$</td>
<td>100</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>.2</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>.3</td>
</tr>
<tr>
<td>$r$</td>
<td>.05</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.05</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>.1</td>
</tr>
</tbody>
</table>

As shown in Table 1, we focus on a case with $\sigma = 0.4$. In line with the empirical evidence, tax rates are assumed to range from 0.2 to 0.3. We also set $r = \delta = 0.05$, which are plausible values, and assume that $\nu = 3$. Given $r = 0.05$, the default cost is therefore about 10% of the debt value.\(^{10}\)

In the following figures we plot the optimal values $C^*$ and $\Pi^*$, which solve problem (9), as a function of parameters $\tau_0$, $\tau_1$, $\lambda$, $\nu$ and $\sigma$. In doing so, we will analyze two different scenarios: the first one where, due to the capital levy problem, the corporate tax rate is expected to rise (i.e., with $\tau_0 < \tau_1$); the second one, where, due to tax competition, the tax rate is expected to be cut (i.e., with $\tau_0 > \tau_1$).

Figure 1A shows the effects of a change in $\tau_0$ on both trigger point $\Pi^*$ and optimal coupon $C^*$. Given $\tau_1$, we can see that both $\Pi^*$ and $C^*$ are always increasing in $\tau_0$. This means that, irrespective of the sign of the tax rate differential ($\tau_1 - \tau_0$), a higher current tax rate discourages entry and raises the tax benefit of debt financing. As we can see, in Figure 1B the ratio $C^*/\Pi^*$ is decreasing in $\tau_0$ and is always higher than the ratio $C/\Pi$ (i.e., the one under tax-rate certainty).

As shown in Figure 2A, an increase in $\tau_1$ always raises $C^*$. This is due to the fact that the higher the rate $\tau_1$, the higher is the expected tax burden faced by a firm, and therefore, the higher is the tax benefit of interest deductibility. This stimulates borrowing.

Moreover, we see that the effect of a change in $\tau_1$ on $\Pi^*$ is ambiguous: it is decreasing (increasing) for low (high) values of $\tau_1$. This result is due to

\(^{10}\)This percentage is in line with Branch’s (2002) estimates.
two offsetting effects. On the one hand, the firm knows that, at any given time, an increase in \( \tau_1 \) means that, in the future, a heavier tax burden will be faced: this discourages entry. On the other hand, the firm is aware that if it invests earlier, it can enjoy a lower tax rate and the tax benefit of interest deductibility for a longer period. Figure 2A shows that the former (latter) effect dominates the latter (former) one, if \( \tau_1 \) is high (low) enough.

It is worth noting that the ratio \( \frac{C}{b} \) is increasing in \( \tau_1 \) and is always higher than the one under tax rate certainty, as shown in Figure 2B. This means that the higher the probability \( \lambda \), the greater is a firm’s incentive to raise leverage. This result may explain the result found by Desai, Foley and Hines (2004), i.e., that policy risk has a positive effect on debt financing.\(^{11}\)

Figures 3-4 shows the effects of a change in the probability of a tax rate change, i.e., \( \lambda \). Let us start with the capital-levy scenario, i.e., with the

\(^{11}\)Such result is also in line with Brealey and Myers (2001), who maintain that debt can be used as a threat against governments aiming to expropriate. In their recommendation to readers who want to set up a mine in the Republic of Costaguana they maintain (p. 810): "No contract can restrain sovereign power. But you can arrange project financing to make these acts as painful as possible for the foreign government. For example, you might set up the mine as a subsidiary corporation, which then borrows a large fraction of the required investment from a consortium of major international banks. If your firm guarantees the loan, make sure the guarantee stands only if the Costaguanan governments honors its contract. The government will be reluctant to break the contract if that causes a default on the loans and undercuts the country’s credit standing with the international banking system".
tax rate differential \((\tau_0 - \tau_1) < 0\). As we can see, the trigger point \(\tilde{\Pi}^*\) is positively affected by \(\lambda\). In other words, the higher the probability of a tax rate increase, the higher is a firm’s trigger point, or equivalently, the later an investment is made. A similar result holds for debt finance: an increase in \(\lambda\) makes the tax rate increase more likely. This implies that the expected tax benefit of interest rate deductibility is increasing in \(\lambda\). Therefore, the higher the parameter \(\lambda\), the more a firm borrows.

As shown in Figure 4, the converse is true if \((\tau_0 - \tau_1) > 0\). Namely, if a tax rate cut is expected to occur, due to competitive pressures, an increase in \(\lambda\) reduces both \(\tilde{\Pi}^*\) and \(C^*\). Let us next analyze the effects of volatility.
As shown in Figure 5A, in a capital-levy scenario, an increase in EBIT volatility raises the firm’s trigger point and reduces the optimal coupon. These effects are similar to those obtained in the absence of tax rate uncertainty. In particular, the decrease in $C^*$ is due to the fact that, with a higher value of $\sigma$, the contingent cost of default rises (see, e.g., Panteghini, 2007b). This discourages debt financing. Accordingly, Figure 5B shows that the ratio $C^*/\hat{\Pi}^*$ is decreasing in $\sigma$. In any case, this ratio is higher than that obtained under tax rate certainty: the difference is substantial for relatively low values of $\sigma$.

As for the trigger point, the quality of results changes when a tax-rate cut is expected. As shown in Figure 6.a, $\hat{\Pi}^*$ is increasing (decreasing) in $\sigma$ for low (high) values of $\sigma$. As usual, for relatively low values of $\sigma$, an increase in $\sigma$ raises the option value, thereby making the opportunity cost of immediate investment higher: this discourages investment. When $\sigma$ is high enough, the option value decreases, thereby encouraging investment.

As for the coupon the result is similar to that obtained in Figure 5.A. As shown in Figures 6.A and 6.B, an increase in $\sigma$ reduces both the optimal coupon and the ratio $C^*/\hat{\Pi}^*$ (Figure 6.B) As usual, $C^*/\hat{\Pi}^*$ is higher than that obtained under tax rate certainty.
Finally, Figure 7 sums up our main findings: trigger point $\hat{\Pi}^*$ is in fact computed as a function of $\Delta \tau = \tau_1 - \tau_0$ for different values of $\lambda$ and in two different cases: when debt finance is not allowed, i.e. $C = 0$, and when it is allowed, i.e. $C = C^*$. The two deriving sets of values of $\hat{\Pi}^*$ are then compared in order to capture the effect of debt finance on the optimal investment timing of a firm. Two remarks are worth highlighting. (i) All values of $\hat{\Pi}^*$ with $C = C^*$ are significantly lower than the corresponding amounts with $C = 0$: this means that debt finance encourages entry. (ii) Slopes of all values of $\hat{\Pi}^*$ with $C = C^*$ are smaller than slopes of the corresponding amounts with $C = 0$: this means that tax rate uncertainty has smaller effect on investment timing decision of a firm, if debt finance is taken into account.
More importantly, the results depicted in Figure 7 show that debt-financed capital structures can alleviate the effects of tax-rate uncertainty on investment decisions. This result holds even under the assumption that debt is non-renegotiable. Of course we expect that, whenever renegotiation is allowed, a firm enjoys a higher degree of flexibility. Therefore, the effect of tax-rate uncertainty is expected to be further mitigated.

4 Conclusion

In this article we have applied a real-option model to study the effects of tax rate uncertainty on both investment timing and the capital structure of a representative firm.

Our two main findings are that debt finance (i) encourages entry and (ii) mitigates the effect of tax rate uncertainty on entry timing.

In this article we have applied many simplifying assumptions, such as the symmetric treatment of profits and losses, as well as the absence of personal taxation, agency costs and any bargaining process between stakeholders (including renegotiation and partial conversion of debt into equity). We believe that the elimination of any of these simplifying assumptions is an interesting topic that we leave for future research.

A The value functions

In order to determine a firm’s value function with tax rate uncertainty we must first calculate the value function after the tax-rate change, i.e. when tax rate is $\tau_1$. We then calculate the value function under tax rate uncertainty, i.e., when the current tax rate is $\tau_0$.

A.1 The value function (4)

Using dynamic programming we first compute the equity value $E_1 (\Pi; C)$ as a summation of the current value in the short interval $dt$ and the remaining value after the instant $dt$ has passed:

$$E_1 (\Pi; C) = (1 - \tau_1) (\Pi - C) dt + e^{-\tau_1 dt} \xi [E_1 (\Pi + d\Pi; C)] ,$$

(10)
where \( \xi \left[ E (\Pi + d\Pi; C) \right] \) is the expected value of equity at time \( t + dt \). Expanding the RHS of (10), applying Itô’s Lemma and rearranging gives the following non-arbitrage condition:

\[
rE_1 (\Pi; C) = (1 - \tau_1) (\Pi - C) + (r - \delta) \Pi E_1_{\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 E_{1\Pi\Pi} (\Pi; C), \tag{11}
\]

where \( \delta \equiv r - \alpha \), \( E_{1\Pi} \equiv \frac{\partial E}{\partial \Pi} \) and \( E_{1\Pi\Pi} \equiv \frac{\partial^2 E}{\partial \Pi^2} \). The general-form solution of (11) is

\[
E_1 (\Pi; C) = \left\{ \begin{array}{ll}
0 & \text{after default,} \\
(1 - \tau_1) \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) + \sum_{i=1}^{2} A_i \Pi^{\beta_i} & \text{before default,}
\end{array} \right. \tag{12}
\]

Terms \( \beta_1 > 1 \) and \( \beta_2 < 0 \) are the roots of the characteristic equation

\[
\Psi(\beta) = \frac{1}{2} \sigma^2 \beta (\beta - 1) + (r - \delta) \beta - r = 0.
\]

These roots are \( \beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \) and \( \beta_2 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \). Let us next calculate the constants \( A_1 \) and \( A_2 \). In the absence of any financial bubbles, \( A_1 \) is nil. To calculate \( A_2 \), notice that default occurs when \( \Pi \) drops to \( C \), namely the condition

\[
E_1 (C; C) = (1 - \tau_1) \left( \frac{C}{\delta} - \frac{C}{r} \right) + A_2 C^{\beta_2} = 0
\]

holds. Solving by \( A_2 \) we get

\[
A_2 = - \left[ (1 - \tau_1) \left( \frac{C}{\delta} - \frac{C}{r} \right) \right] C^{-\beta_2}.
\]

Using this boundary condition we can thus rewrite (12) as

\[
E_1 (\Pi; C) = \left\{ \begin{array}{ll}
0 & \text{after default,} \\
(1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) (\frac{\Pi}{C})^{\beta_2} \right] & \text{before default.} \tag{13}
\end{array} \right.
\]

Following the same procedure we can write the value of debt \( D_1 (\Pi; C) \) as follows:

\[
D_1 (\Pi; C) = C + e^{-r dt} \xi \left[ D_1 (\Pi + d\Pi; C) \right].
\]
Expanding its RHS, applying Itô’s Lemma and rearranging gives the following non-arbitrage condition:

\[
rd_1(\Pi; C) = \begin{cases} 
(1 - \tau_1) \Pi + (r - \delta) \Pi d_1(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 d_{1\Pi}(\Pi; C) & \text{after default,} \\
C + (r - \delta) \Pi d_{1\Pi}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 d_{1\Pi}(\Pi; C) & \text{before default.}
\end{cases}
\]

The closed-form solution of (14) is:

\[
d_1(\Pi; C) = \begin{cases} 
\frac{(1-\tau_1)\Pi}{\delta} + \sum_{i=1}^{2} B_i \Pi^{\beta_i} & \text{after default,} \\
\frac{C}{r} + \sum_{i=1}^{2} D_i \Pi^{\beta_i} & \text{before default.}
\end{cases}
\]

To calculate \( B_2 \) we use the boundary condition \( d_{0}(0; C) = 0 \), which means that when \( \Pi \) falls to zero the lender’s post-default claim is nil. Thus we have \( B_2 = 0 \). In the absence of any financial bubble, moreover, we have \( B_1 = D_1 = 0 \). To calculate \( D_2 \) we let the pre-default branch of (15) meet with its after-default one, net of the default cost \( vC \), at point \( \Pi = C \), i.e.

\[
\frac{C}{r} + D_2 \Pi C^{\beta_2} = \frac{(1 - \tau) C}{\delta} - vC.
\]

Thus we have \( D_2 = \left[ \frac{(1 - \tau_1)C}{\delta} - \frac{C}{r} - vC \right] C^{-\beta_2} \). Using (15) we thus obtain

\[
d_1(\Pi; C) = \begin{cases} 
\frac{(1-\tau_1)\Pi}{\delta} & \text{after default,} \\
\frac{1}{r} + \left( \frac{1 - \tau_1}{\delta} - \frac{1}{r} - v \right) \left( \frac{\Pi}{C} \right)^{\beta_2} C & \text{before default.}
\end{cases}
\]

Summing up the before-default values in (13) and (17) we can calculate the Net Present Value of an investment project:

\[
V_1(\Pi; C) = E_1(\Pi; C) + D_1(\Pi; C) - I \\
= (1 - \tau_1) \left[ \left( \frac{\Pi}{r} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] + \left\{ \frac{1}{r} + \left[ \frac{(1 - \tau_1)}{\delta} - \frac{1}{r} - v \right] \left( \frac{\Pi}{C} \right)^{\beta_2} \right\} C - I \\
= \left( \frac{(1-\tau_1)\Pi}{\delta} + \frac{\tau_1 C}{r} - \left( \frac{\tau_1}{r} + v \right) C \left( \frac{\Pi}{C} \right)^{\beta_2} - I. \right.
\]

(18)
A.2 The value function (5)

Following the same procedure as above we can calculate the value function before the tax rate change, starting with the value of equity $E_0$. Using dynamic programming we can write it as:

$$E_0(\Pi; C) = (1 - \tau_0) (\Pi - C) + (1 - \lambda dt) e^{-r dt} [E_0 (\Pi + d\Pi; C)] + \lambda dt \xi [E_1 (\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô’s Lemma and rearranging gives the following non-arbitrage condition:

$$(r + \lambda) E_0 (\Pi; C) = (1 - \tau_0) (\Pi - C) + (r - \delta) \Pi E_0_{\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 E_{\Pi \Pi} (\Pi; C) + \lambda E_1 (\Pi; C).$$  \hspace{1cm} (19)

Let us next subtract (11) from (19) so that:

$$(r + \lambda) X (\Pi; C) = (\tau_1 - \tau_0) (\Pi - C) + (r - \delta) \Pi X_{\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 X_{\Pi \Pi} (\Pi; C),$$

with

$$X (\Pi; C) \equiv E_0 (\Pi; C) - E_1 (\Pi; C).$$ \hspace{1cm} (20)

Solving (20) gives

$$X (\Pi; C) = (\tau_1 - \tau_0) \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) + \sum_{i=1}^{2} F_i \beta_i (\lambda),$$ \hspace{1cm} (22)

where

$$\beta_1 (\lambda) = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 (r + \lambda)}{\sigma^2}} > 1,$$

$$\beta_2 (\lambda) = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 (r + \lambda)}{\sigma^2}} < 0,$$

are the roots of the characteristic equation

$$\Psi(\beta) = \frac{1}{2} \sigma^2 \beta (\beta - 1) + (r - \delta) \beta - (r + \lambda) = 0.$$
Notice that, in the absence of bubbles we have $F_1 = 0$. Using (21) and (22) we can calculate

$$E_0 (\Pi; C) = E_1 (\Pi; C) + X (\Pi; C)$$

$$= (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right]$$

$$+ (\tau_1 - \tau_0) \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) + F_2 \Pi^{\beta_2(\lambda)}.$$

Using condition $E_0 (C; C)$ we can find

$$F_2 = - (\tau_1 - \tau_0) \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) C^{-\beta_2(\lambda)}.$$

Therefore we obtain:

$$E_0 (\Pi; C) = E_1 (\Pi; C) + X (\Pi; C)$$

$$= (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right]$$

$$+ (\tau_1 - \tau_0) \left[ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \right].$$

We can now calculate the value of debt before the tax rate change:

$$D_0 (\Pi; C) = C + (1 - \lambda dt) e^{-\pi dt} [D_0 (\Pi + d\Pi; C)] + \lambda dt \xi [D_1 (\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô’s Lemma and rearranging gives the following non-arbitrage condition:

$$(r + \lambda) D_0 (\Pi; C) = C + (r - \delta) \Pi D_{000} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 D_{0000} (\Pi; C) + \lambda D_1 (\Pi; C).$$

(24)

Subtracting (14) from (24) gives:

$$(r + \lambda) Y (\Pi; C) = (r - \delta) \Pi Y_{\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 Y_{\Pi\Pi} (\Pi; C),$$

(25)

with

$$Y (\Pi; C) \equiv D_0 (\Pi; C) - D_1 (\Pi; C).$$

(26)
The solution of (25) has the following form:

\[
Y(\Pi; C) = \begin{cases}
\frac{\left(\tau_1 - \tau_0\right)\Pi}{\delta + \lambda} + \sum_{i=1}^{2} L_i \Pi^{\beta_i}\, & \text{after default}, \\
\sum_{i=1}^{2} G_i \Pi^{\beta_i}\, & \text{before default}.
\end{cases}
\]

Notice that, after default (but before the tax rate change), the boundary condition \( Y(0; C) = 0 \) holds. This implies that \( L_2 = 0 \). Moreover, we assume that no bubble exists. This means that \( L_1 = G_1 = 0 \). Let us next write

\[
D_0(\Pi; C) = D_1(\Pi; C) + Y(\Pi; C),
\]

After default (but before the tax rate change) we have:

\[
D_0(\Pi; C) = \left[\frac{1 - \tau_1}{\delta} + \frac{\left(\tau_1 - \tau_0\right)}{\delta + \lambda}\right] \Pi.
\]

Before default we have:

\[
D_0(\Pi; C) = \left\{\frac{1}{r} + \left[\frac{1 - \tau_1}{\delta} - \frac{1}{r - v}\right] \left(\frac{\Pi}{C}\right)^{\beta_2}\right\} C + G_2 \Pi^{\beta_2}.\tag{27}
\]

To find \( G_2 \), we let the two branches of the debt function meet in \( \Pi = C \), i.e.

\[
\left(\frac{1 - \tau_1}{\delta} - v\right) C + G_2 C^{\beta_2} = \left(\frac{1 - \tau_1}{\delta} + \frac{\tau_1 - \tau_0}{\delta + \lambda}\right) C - v C.\tag{28}
\]

Rearranging gives:

\[
G_2 = \frac{\tau_1 - \tau_0}{\delta + \lambda} C^{1 - \beta_2}\tag{29}.
\]

Substituting in (27)

\[
D_0(\Pi; C) = \left[\frac{1}{r} + \left(\frac{1 - \tau_1}{\delta} - \frac{1}{r - v}\right) \left(\frac{\Pi}{C}\right)^{\beta_2}\right] C + \frac{\tau_1 - \tau_0}{\delta + \lambda} C \left(\frac{\Pi}{C}\right)^{\beta_2}.\tag{29}
\]
Using (23) and (29) we finally obtain the value function $V_0$:

$$V_0(\Pi; C) = (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{\tau} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] +$$

$$+ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \tau} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} +$$

$$+ \left( \frac{1 - \tau_1}{\delta} - \frac{1}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} C + \frac{\tau_1 - \tau_0}{\delta + \lambda} C \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} - I. \tag{30}$$

B     The option functions

In order to address a firm’s strategy we first elaborate its strategy under tax rate certainty, when tax rate is equal to $\tau_1$. We then calculate the firm’s option value under tax rate uncertainty, i.e., when the tax rate is $\tau_0$.

B.1     The option function (6)

The firm is endowed with an option to invest. Using dynamic programming we can write a firm’s option to invest under tax rate certainty as:

$$O_1(\Pi; C) = e^{-r dt} \xi \left[ O_1(\Pi + d\Pi; C) \right].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$rO_1(\Pi; C) = (r - \delta) \Pi O_1_{\Pi}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 O_1_{\Pi^2}(\Pi; C). \tag{31}$$

As shown in Dixit and Pindyck (1994), the general-form solution of (31) is:

$$O_1(\Pi; C) = H_1 \Pi^{\beta_1},$$

where $H_1$ is an unknown. To calculate $H_1$, we apply the VMC at the entry threshold level

$$V_1(\Pi; C)|_{\Pi = \Pi} = O_1(\Pi; C)|_{\Pi = \Pi},$$

21
so as to find

\[ H_1 = V_1(\Pi; C)\Pi^{-\beta_1} = \]

\[ = \left[ \frac{(1 - \tau_1) \Pi}{\delta} + \frac{\tau_1 C}{r} - \left( \tau_1 + rv \right) \frac{C}{r} \left( \frac{\Pi}{C} \right)^{\beta_2} - I \right] \Pi^{-\beta_1} \]  \hspace{2cm} (32)

Therefore we can write the option function as:

\[ O_1 (\Pi, C; \Pi) = \left( \frac{\Pi}{\Pi} \right)^{\beta_1} \left[ \frac{(1 - \tau_1) \Pi}{\delta} + \frac{\tau_1 C}{r} - \left( \tau_1 + rv \right) \frac{C}{r} \left( \frac{\Pi}{C} \right)^{\beta_2} - I \right]. \]  \hspace{2cm} (33)

### B.2 The option function (7)

Using dynamic programming we can write a firm’s option to invest under tax rate uncertainty as:

\[ O_0 (\Pi; C) = (1 - \lambda dt) e^{-r dt} \xi \left[ O_0 (\Pi + d\Pi; C) \right] + \lambda dt \xi \left[ O_1 (\Pi + d\Pi; C) \right]. \]

Expanding its RHS, applying Itô’s Lemma and rearranging gives the following non-arbitrage condition:

\[ (r + \lambda) O_0 (\Pi; C) = (r - \delta) \Pi O_{0\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 O_{0\Pi} (\Pi; C) + \lambda O_1 (\Pi; C). \]

Subtracting (31) from (34) gives

\[ (r + \lambda) Z (\Pi; C) = (r - \delta) \Pi Z_{0\Pi} (\Pi; C) + \frac{\sigma^2}{2} \Pi^2 Z_{0\Pi} (\Pi; C), \]  \hspace{2cm} (35)

where \( Z (\Pi; C) \equiv O_0 (\Pi; C) - O_1 (\Pi; C). \) Solving we have:

\[ Z (\Pi; C) = \sum_{i=1}^{2} Z_i \Pi^{\beta_i(\lambda)}. \]

Since \( Z (0; C) = 0, \) we have

\[ Z (\Pi; C) = Z_1 \Pi^{\beta_1(\lambda)}. \]
Therefore we obtain:

\[ O_0 (\Pi; C) = O_1 (\Pi; C) + Z (\Pi; C) = H_1 \Pi^{\beta_1} + Z_1 \Pi^{\beta_1 (\lambda)}. \]  

To compute \( Z_1 \) we use the VMC at the threshold point \( \Pi = \hat{\Pi} \):

\[ V_0 (\Pi; C) |_{\Pi = \hat{\Pi}} = O_0 (\Pi; C) |_{\Pi = \hat{\Pi}}. \]  

Using (36) and (37) we can write:

\[ H_1 \hat{\Pi}^{\beta_1} + Z_1 \hat{\Pi}^{\beta_1 (\lambda)} = V_0 \left( \hat{\Pi}; C \right), \]

which gives

\[ Z_1 = \left[ V_0 \left( \hat{\Pi}; C \right) - H_1 \hat{\Pi}^{\beta_1} \right] \hat{\Pi}^{-\beta_1 (\lambda)}. \]

Substituting in \( Z (\Pi; C) = Z_1 \Pi^{\beta_1 (\lambda)} \) the value of \( H_1 \) as in (32) one gets:

\[ Z_1 \Pi^{\beta_1 (\lambda)} = \left[ V_0 \left( \hat{\Pi}; C \right) - H_1 \hat{\Pi}^{\beta_1} \right] \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1 (\lambda)}. \]

Given \( H_1 = V_1 (\Pi; C) \hat{\Pi}^{-\beta_1} \) we can write:

\[ O_0 (\Pi; C) = H_1 \Pi^{\beta_1} + Z_1 \Pi^{\beta_1 (\lambda)} \]

\[ = V_1 (\Pi; C) \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} + \left[ V_0 \left( \hat{\Pi}; C \right) - V_1 (\Pi; C) \left( \frac{\hat{\Pi}}{\Pi} \right)^{\beta_1} \right] \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1 (\lambda)}. \]

We can therefore write the problem (9).

C Optimal strategies

C.1 Tax rate certainty

By solving (8), a firm chooses both its entry timing and its capital structure when tax rate is certain and equal to \( \tau_0 \). First order condition w.r.t. \( C \) is

\[ \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} \frac{1}{r} \left[ \tau_0 - (1 - \beta_2) (\tau_0 + vr) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] = 0. \]  

23
Rearranging (39) one easily obtains

\[ C = \left( \frac{1}{1 - \beta_2} \frac{\tau_0}{r \tau_0 + rv} \right)^{-\frac{1}{\beta_2}} \tilde{\Pi}. \]  

(40)

First order condition w.r.t. \( \tilde{\Pi} \) is

\[
\left( \frac{\Pi}{\tilde{\Pi}} \right)^{\beta_1} \frac{1}{\tilde{\Pi}} \left\{ \frac{(1-\tau_0)\tilde{\Pi}}{r} - \beta_2 \frac{C}{r} (\tau_0 + vr) \left( \frac{\tilde{\Pi}}{C} \right)^{\beta_2} + -\beta_1 \left[ \frac{(1-\tau_0)\tilde{\Pi}}{r} + \frac{C}{r} \left( \tau_0 - (\tau_0 + vr) \left( \frac{\tilde{\Pi}}{C} \right)^{\beta_2} \right) - I \right] \right\} = 0. \]  

(41)

From (39) we have

\[
\frac{\tau_0}{1 - \beta_2} = (\tau_0 + vr) \left( \frac{\tilde{\Pi}}{C} \right)^{\beta_2}.
\]

We can hence rewrite (41) as

\[
\left[ \frac{(1 - \tau_0)\tilde{\Pi}}{r} - \beta_2 \frac{C}{r} \frac{\tau_0}{1 - \beta_2} \right] - \beta_1 \left[ \frac{(1 - \tau_0)\tilde{\Pi}}{r} + \frac{C}{r} \left( \tau_0 - \frac{\tau_0}{1 - \beta_2} \right) - I \right] = 0.
\]

Rearranging

\[
(1 - \beta_1) \frac{(1 - \tau_0)\tilde{\Pi}}{r} + (1 - \beta_1) \frac{\tau_0}{r} \frac{C}{\beta_2} \frac{\beta_2}{1 - \beta_2 - 1} + \beta_1 I = 0.
\]

Dividing by \( \frac{(1-\beta_1)(1-\tau_0)}{r} \)

\[
\tilde{\Pi} + \frac{\tau_0}{1 - \tau_0} \frac{\beta_2}{\beta_1 - 1} \frac{C}{1 - \tau_0} - \frac{\beta_1}{\beta_1 - 1} \frac{r}{1 - \tau_0} I = 0.
\]

Substituting in the above expression the value of \( C \) as in (40), one obtains

\[
\tilde{\Pi} = \frac{1}{1 + m} \frac{\beta_1}{\beta_1 - 1} \frac{r}{1 - \tau_0} I,
\]

where

\[
m \equiv \frac{\tau_0}{1 - \tau_0} \frac{\beta_2}{\beta_2 - 1} \left( \frac{1}{1 - \beta_2} \frac{\tau_0}{\tau_0 + vr} \right)^{-\frac{1}{\beta_2}} > 0.
\]
The following system sums up a firm’s optimal entry timing and coupon, denoted with $\bar{\Pi}^*$ and $\bar{C}$ respectively, when tax rate is certain and equal to $\tau_0$:

$$
\begin{align*}
\bar{\Pi}^* &= \frac{1}{1+m} \frac{\beta_1}{\beta_1 - 1} \frac{r I}{1 - \tau_0}, \\
\bar{C} &= \left( \frac{1}{1 - \beta_2 \tau_0 + r v} \right)^{-\frac{1}{\beta_2}} \bar{\Pi}^*.
\end{align*}
$$

It is useful to provide some comparative statics result on $\bar{\Pi}^*$ and $\bar{C}$. The derivative of $\bar{\Pi}^*$ w.r.t. $\tau_0$ is

$$
\frac{\partial \bar{\Pi}^*}{\partial \tau_0} = \frac{\beta_1}{\beta_1 - 1} \frac{r I}{(1 - \tau_0)^2} \frac{1 + m - \frac{\partial m}{\partial \tau_0} (1 - \tau_0)}{(1 + m)^2}.
$$

(42)

The derivative of $\bar{C}$ w.r.t. $\tau_0$ is

$$
\frac{\partial \bar{C}}{\partial \tau_0} = \left( \frac{(1 - \beta_2) (\tau_0 + r v)}{\tau_0} \right)^{-\frac{1}{\beta_2}} \left( \frac{\partial \bar{\Pi}^*}{\partial \tau_0} - \frac{1}{\beta_2 \tau_0 + r v} \bar{\Pi}^* \right).
$$

By differentiating $m$ w.r.t. $\tau_0$ we get

$$
\frac{\partial m}{\partial \tau_0} = \frac{\beta_2}{\beta_2 - 1} \left( \frac{1}{1 - \beta_2} \right)^{-\frac{1}{\beta_2}} \left( 1 + \frac{v r}{\tau_0} \right)^{-\frac{1}{\beta_2}} \frac{r v (\tau_0 + v r) \beta_2 - v r (1 - \tau_0)}{(1 - \tau_0)^2 \beta_2 (\tau_0 + v r)}.
$$

It is easy to check that all the above terms are strictly positive, hence $\frac{\partial m}{\partial \tau_0} > 0$.

From (42)

$$
\frac{\partial \bar{\Pi}^*}{\partial \tau_0} = \frac{\beta_1}{\beta_1 - 1} \frac{r I}{(1 - \tau_0)^2} \frac{1 + m - \frac{\partial m}{\partial \tau_0} (1 - \tau_0)}{(1 + m)^2}
$$

we know that if $\frac{\partial m}{\partial \tau_0} < \frac{1 + m}{1 - \tau_0}$, then $\frac{\partial \bar{\Pi}^*}{\partial \tau_0} > 0$. The inequality $\frac{\partial m}{\partial \tau_0} < \frac{1 + m}{1 - \tau_0}$ can be rewritten as

$$
\frac{\beta_2 (\tau_0 + v r) - v r}{(\beta_2 - 1) (\tau_0 + v r)} \left( \frac{1}{\tau_0 + v r} \right)^{-\frac{1}{\beta_2}} < 1.
$$

Notice that both $\frac{\beta_2 (\tau_0 + v r) - v r}{(\beta_2 - 1) (\tau_0 + v r)}$ and $\left( \frac{\tau_0}{(1 - \beta_2) (\tau_0 + v r)} \right)^{-\frac{1}{\beta_2}}$ are less than 1. It follows that $0 < \frac{\partial m}{\partial \tau_0} < \frac{1 + m}{1 - \tau_0}$. We can conclude that $\frac{\partial \bar{\Pi}^*}{\partial \tau_0} > 0$. Moreover

$$
\frac{\partial \bar{C}}{\partial \tau_0} = \left( \frac{(1 - \beta_2) (\tau_0 + r v)}{\tau_0} \right)^{-\frac{1}{\beta_2}} \left( \frac{\beta_2 - 1}{\beta_2 \tau_0 + r v} \bar{\Pi}^* + \frac{\partial \bar{\Pi}^*}{\partial \tau_0} \right) > 0.
$$

if $\frac{\partial \bar{\Pi}^*}{\partial \tau_0} > 0$ since all the other terms are positive.
C.2 Tax rate uncertainty

Using (38) we can rewrite problem (9) as follows:

\[
\max_{\hat{\Pi} > 0, C > 0} O_0 (\hat{\Pi}, C; \Pi) = \max_{\hat{\Pi} > 0, C > 0} \left\{ \frac{V_1(\hat{\Pi}; C)}{\hat{\Pi}} \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1} + \right. \\
\left. \left[ V_0 (\hat{\Pi}; C) - V_1(\hat{\Pi}; C) \left( \frac{\hat{\Pi}}{\Pi} \right)^{\beta_1} \right] \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda)} \right\}.
\]

The first order conditions are therefore equal to:

\[
\frac{\partial O_0 (\Pi; \hat{\Pi}; C)}{\partial \hat{\Pi}} = V_1(\hat{\Pi}; C) \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1} - \frac{\beta_1(\lambda)}{\Pi} V_0 (\hat{\Pi}; C) \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda)} = 0.
\]

and

\[
\frac{\partial V_0 (\hat{\Pi}; C)}{\partial \hat{\Pi}} \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda)} = 0.
\]

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