REPUTATION AND COMPETITION
WITH SOCIAL CONVENTION

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Discussion Paper n. 1105

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Reputation and Competition with Social Convention*

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November 2011

Abstract

In this short paper we develop a pure hidden action model of reputation and repeated Bertrand competition, where firms are homogeneous, entry is free and consumers infer future non-contractile quality both from past quality and from current prices. We show that a positive level of quality is sustained as an outcome of a stationary equilibrium, together with a social convention about the minimum acceptable quality.

Keywords: nonobservable quality, pure hidden action, reputation, Bertrand competition, social convention.

JEL Codes: L13, L14, D82, C73.

1 Introduction

Reputation is a crucial aspect when quality is hard to measure. The seminal paper by Klein and Leffler (1981) shows that in on-going relationships clients can react to a monopolistic firm’s choice of providing low quality by not repeating their purchase. This reaction constitutes a punishment for the firm because providing high quality commands a quality premium. Shapiro (1983) and Allen (1984) extend the analysis to a competitive setup and prove that the quality premium is just sufficient to cover the higher costs of quality. However, the two contributions rely on non fully rational consumers’ expectations. In Shapiro (1983), for instance, consumers infer future quality only from the observation of past levels: since they underestimate quality of new goods, firms are obliged to signal high quality through low initial prices. This mechanism would disappear if consumers were more rational, i.e. if they inferred future quality also from current prices. Indeed, a firm could enter the market adopting a non-stationary strategy: reducing quality and the short run price so that consumers are better-off and, at the same time, the firm long run incentive to produce high quality is preserved. Rational consumers would thus be convinced about the quality of its product and the undercutting strategy would be profitable, since the firm might recover profits by raising slightly the future price. This reasoning leads to the famous Stiglitz (1989) objection: competition should eliminate the quality premium, making reputation unable to operate and to induce the production of high-quality goods. Solution to the Stiglitz problem came from Hörner (2002).

Before describing Hörner’s results and introducing ours, we provide a brief account of literature on reputation. According to Bar-Isaac and Tadelis (2008) wording, models of reputation can be classified in three main categories: (i) pure hidden action, where only one type of seller is present in the market,
who can provide goods of different quality; (ii) pure hidden information, where sellers of different types have no control over quality of the product; (iii) mixed frameworks, which include both hidden action and hidden information. Klein and Leffler (1981), Shapiro (1983) and Allen (1984) fall in the first class, where the Stiglitz problem appears to be more severe, unless not fully rational expectations are considered. For instance, Tadelis (2002) and Toth (2008) condition quality expectations only to past quality level and not to current prices. On the contrary, Kranton (2003) assumes fully rational beliefs and confirms Stiglitz objection by demonstrating that a firm can gain by cutting its price, absent collusion. Dana and Fong (2010) provide a model of reputation and collusion. They show that a relatively concentrated market structure better sustains high quality, since firms are punished by rivals when lowering price and by consumers when cutting quality.\footnote{In a model with both vertical and horizontal differentiation, Bar-Isaac (2005) demonstrates instead that high quality can be sustained in markets where the degree of product substitutability is either low or high.} Finally, Hörner (2002) adopts a mixed model. He considers good and bad firms: the former have a technological advantage in producing high quality. Quality is also affected by firms’ effort choice and some randomness in a repeated market interaction. At equilibrium all firms who underperformed in quality are kicked out of the market, good firms are induced to invest in quality to avoid being pushed out of the market and there might be positive profit: hence reputation is valuable.

In this short paper we introduce a moral hazard model of reputation and (Bertrand) competition where homogeneous consumers and firms interact repeatedly and entry is free. Unlike most existing literature, our focus is on services instead of physical goods.\footnote{Since capacity constraints are less relevant in the provision of services, the assumption of Bertrand competition seems to be appropriate.} This makes an important difference: product quality is determined before or simultaneously to prices or quantities, while service quality is determined later; only the latter timing is fully compatible with post-contractual informational asymmetry. We assume that consumers infer future noncontractible quality both from past quality and from current prices. More exactly, consumers are able to compute the firms’ incentive compatibility (IC henceforth) constraint, which is based on the following mechanism: cheating (i.e. producing a lower than promised quality level) entails the cost of losing market share in the future, because we assume the existence of an imperfect public signal about firms’ quality. The public signal can be the result of word of mouth communication or originated by specialized publications or, finally, by forums and discussion groups on the internet.\footnote{The Ebay system of feedbacks, i.e. the ex-post evaluation of sellers (and buyers) made by the counterpart, is a real-world example of the public signal we have in mind; Trip advisor is another one.}

We prove the existence of a stationary equilibrium with positive profits and high quality: to the best of our knowledge no other paper overcame the Stiglitz objection in a pure hidden action model with fully rational consumers. In order for our argument to go through, we have to consider an equilibrium which implies the existence of a social convention about the minimum acceptable quality. Such a convention is an equilibrium result and it represents the main novelty of our analysis: it prevents low quality levels from being accepted and blocks further entry.\footnote{Market shares of online insurance companies, which generally offer lower quality than traditional competitors, experienced very little growth in many economies since their appearance in the market (see, e.g., Brand, 2000, for the UK case): this is an illustration of the social convention we have in mind.} Indeed, the equilibrium quality level increases with the firms’ market share, hence potential new entrants would provide a lower quality than the minimum socially accepted. Since this minimal quality level blocks entry, profits cannot be nought in equilibrium. This explains our main result: even if firms compete à la Bertrand, they end up with equilibrium positive profits, which enable to sustain the investment in high quality. If, instead, the social convention is not part of the equilibrium or the socially accepted quality is extremely low, then more entry is compatible with the equilibrium: consumer
and social welfare are then shown to diminish. This argument explains a second important result: more concentrated industry structures can generate a higher consumer and social welfare. This finding has relevant policy implications: it should induce cautiousness in advocating that competition has to be enhanced when quality is at stake. Finally, we provide a new rationale for the role of social convention. Established views (see, e.g., Kandori, 1992) point to the relevance of social norms as an informal enforcement mechanism to support efficient outcomes in infrequent transactions. We prove that they might be crucial also when economic actors interact frequently.

The remainder of the paper is organized as follows. Section 2 studies the equilibrium of Bertrand competition among the firms when service quality is observable. In Section 3 we relax the observability assumption. In Section 4 we analyze the reputational issue. Section 5 discusses the results and concludes. All proofs are in the Appendix.

2 Competition with Observable Quality

We introduce an economy with a continuum of homogeneous consumers of measure one and \( n \geq 2 \) homogeneous firms that provide a service. Each consumer buys at most one unit of the service, in which case she is characterized by the following utility function: \( u \equiv \alpha q_i - p_i \geq 0 \), where \( q_i \) and \( p_i \) are quality level and price, respectively, of the service supplied by firm \( i = 1, \ldots, n \), and \( \alpha > 0 \) is a parameter capturing the marginal taste for quality; \( \alpha \) is equal across all consumers. Firm \( i \) is characterized by the following profit function:

\[
\Pi_i = \sigma_i [p_i - c(q_i)],
\]

where \( \sigma_i \leq 1 \) denotes the fraction of consumers served by firm \( i \) and \( c(q_i) \) the unitary cost of quality \( q_i \), with \( c(q_i) \) twice differentiable, \( c' > 0 \) and \( c'' > 0 \).

In this section we suppose that service quality \( q_i \) is observable, therefore also contractible. We define the first best contract as the solution to the following problem: a representative consumer maximizes her utility \( u \) subject to firm \( i \)'s participation constraint \( \Pi_i \geq 0 \). We show that the equilibrium of a one-shot Bertrand competition game among the firms replicates the first best.

The timing of the Bertrand game is as follows: (i) firms compete à la Bertrand by making simultaneous offers of \( q \) and \( p \); (ii) each consumer either selects the preferred couple or refuses to purchase; (iii) the accepted contracts are implemented.

**Lemma 1** The Bertrand equilibrium contract when quality is observable has the following features: (i) the firms get zero profits; (ii) the level of service quality is positive; (iii) the consumers obtain maximum utility. In symbols:

\[
\begin{align*}
    p_{FB} &= c(q_{FB}), \\
    c'(q_{FB}) &= \alpha.
\end{align*}
\]

Contract (1) maximizes unitary welfare, defined as the sum of consumer utility and firm per-contract profits:

\[
q_{FB} = \arg \max \left( u + \frac{\Pi_i}{\sigma_i} = \alpha q_i - c(q_i) \right).
\]

3 Competition with Unobservable Quality

In this section we relax the assumption of observability of quality.\(^5\) This means that contracts cannot be conditioned on \( q \). As a consequence, in a one-shot relationship firm \( i \) has an incentive to renege a promised

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\(^5\)In Section 5 we discuss the less restrictive case of fully observable but nonverifiable quality.
high quality level after signing a contract \( \{ q_i, p_i \} \). Indeed, \( \Pi_i \) is decreasing in \( q_i \) for any \( p_i \): the minimum possible level of \( q_i \) will be the profit-maximizing choice. We denote such a level with \( q \geq 0 \) and define it as a minimum legal standard, or a level below which under-provision of quality can be easily verified by a Court.

We replicate the analysis of Section 2 by studying a one-shot Bertrand competition game when \( q = \frac{q}{2} \). The time structure of the game is as in Section 2.

**Lemma 2** The Bertrand equilibrium contract when quality is nonobservable has the following features: (i) the firms get zero profits; (ii) the level of service quality is minimal; (iii) the consumers obtain the maximum utility subject to \( q = \frac{q}{2} \). In symbols:

\[
\begin{cases}
    p = c(q), \\
    q = q \frac{1}{2}.
\end{cases}
\]

(2)

We compare contract (2) to contract (1): firms’ profits are nil under both arrangements, but the latter maximizes unitary welfare \( \alpha q_i - c(q_i) \). It follows that the clients’ utility is lower under contract (2): the equilibrium contract when quality is nonobservable entails unexploited gains from trade. For ease of exposition we let \( c(q) = 0 \): this is without loss of generality, as one can easily check.

4 Contracting and Competition with Reputation

In this section we investigate whether reputation helps reducing the inefficiency due to nonobservability of quality. We first study the contracting problem between a representative firm and its customers; we then extend the analysis to competition.

4.1 Contracting

We consider a repeated interaction among infinitely lived consumers and firms. We denote with \( \sigma_{i,t} \) the fraction of consumers served by firm \( i \) at time \( t = 0, \ldots, \infty \). In each period \( t \geq 0 \), contracting between firm \( i \) and its customers takes place according to the following timing:

- a representative consumer offers a contract \( \{ q_{i,t}, p_{i,t} \} \) to firm \( i \);
- firm \( i \) either accepts the contract or refuses it;
- firm \( i \) selects a quality level \( q_{i,t}^A \) for each consumer, where superscript \( A \) stands for actual;\(^6\)
- in each period Nature selects the following public signal: with probability \( \varphi (\tau_{i,t}, \sigma_{i,t}) \in [0,1] \) all consumers receive a signal of bad quality, where \( \tau_{i,t} \) is the share of clients who enjoy a quality level \( q_{i,t}^A \) lower than the promised \( q_{i,t} \), i.e. the cheated consumers;
- if the consumers receive a signal of bad quality, they know that firm \( i \) cheated somebody; they then decide whether to renew the contract or not.

The above timing depicts a moral hazard model, where the hidden effort is the actual level of \( q_i \) provided by firm \( i \) after the contract is signed. We introduce the following

\(^6\)A focus on physical goods instead of services would make implausible the fact that a specific level of quality is chosen for each consumer.
Assumption 1 (i) $\varphi (0, \sigma_{i,t}) = 0$: if firm $i$ cheats no consumers, no signal is conveyed, i.e. we rule out the possibility that a non-cheated customer sends a signal of bad quality; (ii) $\varphi_{\tau} (\tau_{i,t}, \sigma_{i,t}) > 0$ and $\varphi_{\tau\tau} (\tau_{i,t}, \sigma_{i,t}) \geq 0$, where subscripts $\tau$ (and $\sigma$ below) denote partial derivatives: probability $\varphi$ is increasing and nonconcave in the fraction $\tau$ of cheated consumers; (iii) $\varphi_{\tau\sigma} (\tau_{i,t}, \sigma_{i,t}) > 0$: this inequality has a simple interpretation which we discuss below.

At time $t$ the discounted value of firm $i$’s profit is

$$V_{i,t} \equiv \sigma_{i,t} \left[ p_{i,t} - c(q_{i,t}) + \tau_{i,t} (c(q_{i,t}) - c(q_{i,t}^A)) \right] + \delta [1 - \varphi (\tau_{i,t}, \sigma_{i,t})] V_{i,t+1},$$

where $\delta$ is the discount factor. A trade-off is faced now by firm $i$: when cheating $\tau_{i,t}\sigma_{i,t}$ consumers at any time $t \geq 0$, firm $i$ saves on costs, but it incurs an expected loss of future profits, provided that no client renews the contract when receiving the signal of bad quality. We will verify that this is an equilibrium behavior.

In this context, Lemma 3 computes parametric conditions for which the firms find it profitable not to cheat any customer, i.e. $\tau_{i,t} = 0$ for any firm $i$ at any time $t$: this defines the firms’ IC constraint.

Lemma 3 In a stationary strategy, firm $i$ decides to cheat no consumers if and only if its market share is relatively high, i.e. $\sigma_i \geq \sigma_j$, where $\sigma_j$ satisfies with equality the following condition:

$$\varphi_{\tau} (0, \sigma_i) \geq \frac{1 - \delta}{\delta} \frac{c(q_i)}{p_i - c(q_i)}.$$

The result of Lemma 3 relies on Inequality (iii) of Assumption 1, $\varphi_{\tau\sigma} (\tau, \sigma) > 0$, which amounts to require that, as the market share of firms rises, the probability the consumers receive a signal of bad quality becomes more sensitive to the number of cheated clients. Therefore, a bigger firm finds it less profitable to cheat any customer in order to save on administrative costs for such a positive effect on $V_{i,t}$ is likely to be outdone by the negative one due to a large probability that no consumer will renew the contract.\footnote{Condition $\varphi_{\tau\sigma} (\tau, \sigma) > 0$ is in the spirit, e.g., of Fishman and Rob (2005), who show that word of mouth reputation is valued more by larger firms.}

We are now able to compute the optimal contract with reputation as a solution to the following problem: since the model is stationary a representative consumer maximizes her single-period utility $u$ subject to firm $i$’s participation and IC constraints, $\Pi_i \geq 0$ and (3), respectively. Before proceeding, note that (3) can be rearranged as

$$\Pi_i \equiv p_i - c(q_i) \geq \frac{1 - \delta}{\delta} \frac{c(q_i)}{\varphi_{\tau} (0, \sigma_i)},$$

which implies positive profits for firm $i$, hence assuring its participation.

Lemma 4 The optimal contract with reputation when quality is nonobservable has the following features: (i) the IC constraint (3) is binding, hence the firms get positive profits; (ii) the level of service quality is positive; (iii) the consumers obtain maximum utility given the firms’ IC constraint. In symbols:

$$\left\{ \begin{array}{l}
\ p_r = \kappa (\sigma_i) \times c(q_r), \\
\ c'(q_r) = \frac{\delta}{\sigma (\sigma_i)},
\end{array} \right. \quad (4)$$

where $\kappa (\sigma_i) \equiv \frac{\delta \varphi_{\tau} (0, \sigma_i) + 1 - \delta}{\delta \varphi_{\tau} (0, \sigma_i)} \geq 1$, with $\frac{\partial \kappa (\sigma_i)}{\partial \sigma_i} < 0$. Moreover, the consumer utility in (4) increases with $\sigma_i$.\footnote{Since $u_i$ in (4) is increasing in $\sigma_i$, monopoly is the optimal market structure, i.e. $\sigma_i = 1$, in which case giving full bargaining power to the consumers seems not to be reasonable. Yet, here we simply build up an appropriate benchmark for the subsequent analysis of Bertrand competition.}
Contract (4) represents a Pareto improvement with respect to contract (2), thus reducing the magnitude of unexploited gains from trade discussed at the end of Section 3. Indeed, (2) satisfies the firm IC constraint at \( q = \bar{q} \). Yet, the consumers do not select it when reputation can be built up: this means that they are better off when choosing contract (4). Also the firms are, since they end up with positive profits instead of zero. In addition, the consumers’ utility augments in a more concentrated market for the LHS of (3), binding at equilibrium, is increasing in \( \sigma_i \); on the contrary, the RHS increases with \( q_i \) and decreases with \( p_i \), ceteris paribus. It follows that as \( \sigma_i \) augments firm \( i \) finds it profitable not to cheat even for bigger \( q_i \) and/or smaller \( p_i \).

Before proceeding with the analysis of competition, we study how quality is affected by market share \( \sigma_i \) at the optimum described by (4).

**Lemma 5** Quality level \( q_r \) increases with market share \( \sigma_i \).

Also the above result hinges upon condition \( \varphi_{r\sigma}(\tau, \sigma) > 0 \), thanks to which the IC constraint (3) holds for relatively high \( \sigma_i \). Accordingly, only firms with higher market share can credibly offer higher quality.

### 4.2 Competition

In this subsection we introduce competition by studying the following infinitely repeated game with free entry:

1. firms decide whether to enter the market;
2. firms compete à la Bertrand on \( p_t, q_t \) and \( \tau_t \); the level of quality \( q_t \) is nonobservable, contrary to all of the other variables, hence it is just promised by firms;
3. consumers either select the preferred contract or do not purchase;
4. firms select an actual level of \( q_t \) for each consumer;
5. Nature selects the public signal;
6. the game starts again from stage a.

We solve the game by focusing on symmetric Subgame Perfect Equilibrium (SPE, henceforth) in pure strategies. Symmetry means that all the firms choose the same market share, i.e. \( \sigma_i = 1/n \) for any \( i \).

**Proposition 1** A SPE of the infinitely repeated game described above has the following features:

1. the equilibrium number of firms is
   \[
   n_{SB} = \max \{ n : q_r(n) \geq \bar{q} > q \} : 
   \]
   \( \bar{q} \) is a threshold quality level whose role is specified below, and \( n_{SB} \) must be weakly higher than 2. Entry in each period restores the appropriate number of firms;
2. at equilibrium all firms offer a contract characterized by:
   \[
   \begin{align*}
   p_{SB} &= \kappa(n_{SB}) \times c(q_{SB}) \times \frac{\alpha}{\kappa(n_{SB})}; \\
   c'(q_{SB}) &= \frac{\alpha}{\kappa(n_{SB})};
   \end{align*}
   \]
3. out of equilibrium, that is if \( n > n_{SB} \), all firms offer the contract of Lemma 2:
\[
\begin{align*}
\begin{cases}
p = c(q) , \\q = \tilde{q} ;
\end{cases}
\end{align*}
\] (7)

4. consumers accept contract (6) if \( n \leq n_{SB} \), and accept contract (7) otherwise; they refuse any other contract;

5. consumers refuse any contract from firm \( i \) after receiving the public signal, in which case firm \( i \) exits the market.

Equilibrium contract (6) is equivalent to (4) with \( \sigma_i = 1/n_{SB} \): quality is thus positive and the number of firms is finite. The latter result implies the former and it is driven by consumer implicit expectations, an example of which is given by the following system:
\[
\Pr (i \text{ cheats } | p_i, q_i, n_i) = \begin{cases} 0 & \text{if } \sigma_i \geq \sigma_i \text{ and } q_i \geq \tilde{q} \\ 1 & \text{otherwise;}
\end{cases}
\] (8)

According to (8), the consumers anticipate that firm \( i \) will not cheat when offering \((p_i, q_i)\) if the contract satisfies the IC constraint for any given \( \sigma_i \) and provides quality weakly higher than \( \tilde{q} \). Level \( \tilde{q} \) can be interpreted as a socially accepted quality standard. On the contrary, if (3) is not satisfied and/or firm \( i \) offers less than \( \tilde{q} \), the consumers believe that firm \( i \) wants to cheat all of them. The reason for that is clear when the incentive constraint is not satisfied, while an explanation is needed in the latter case. If firm \( i \) promises \( q_i < \tilde{q} \), whilst any other competitor is fulfilling the socially accepted quality standard by offering (6), each individual consumer, in conformity with the equilibrium strategy, expects that none of the current clients will accept the contract proposed by firm \( i \); she thus anticipates that firm \( i \)'s market share will lower significantly, with the effect that the LHS of (3) decreases given \( \varphi_{\tau q} > 0 \), and that the IC constraint will be satisfied only for \( q \) close to \( \tilde{q} \); in this case she turns to any other competitor. The social standard for quality and comparative statics of Lemma 5 drive the result on the equilibrium number of firms, \( n_{SB} \).

Finally, Proposition 1 implies that competition decreases consumer and social welfare. The effect on consumer welfare stems directly from the fact that at the equilibrium contract (6), the IC constraint (3) is binding: we then know from Lemma 4 that consumers are worse-off in a competitive environment. The same holds true for the firms. Indeed, if social standard for quality, \( \overline{q} \), decreases and approximates \( \tilde{q} \), more firms enter given (5): IC constraint (3) is then satisfied only for \( q \) close to \( \tilde{q} \) and equilibrium profits tends to zero according to Lemma 2.

## 5 Concluding Remarks

Before summing up our findings, we discuss two issues that could be problematic to our analysis: (i) uniqueness and (ii) robustness of the equilibrium. (i) Contract (6) depends on the implicit expectations (8). Consumers’ expectations can sustain different equilibria. Even limiting the analysis to the proposed expectations, distinct equilibria are compatible with a diverse social standard for quality, \( \overline{q} \), which is not uniquely determined in our equilibrium. (ii) Our results depends crucially on condition \( \varphi_{\tau q} > 0 \). If we let \( \varphi_{\tau q} < 0 \), the IC constraint (3) rewrites as \( \sigma_i \leq \sigma_i \), which is not violated by entry of new firms; this means that at the equilibrium, where the IC constraint is binding, contracts entail \( \sigma_i \rightarrow 0 \) and, given \( \varphi_{\tau q} < 0 \), positive per-contract profits and quality. Another important assumption is that quality cannot be fully
observed ex-post. We can relax it by introducing private signals, according to which consumers always leave a firm after observing low quality. The characterization of the equilibrium with purely private signals, when it exists, yields qualitatively similar results, provided that there is a social convention: quality is fixed at the social convention level and the price is set so that firms’ IC constraint is binding. However, per-contract profits do not depend on the market share, hence free entry implies $\sigma_i \to 0$ and the number of firms is indeterminate, unless we assume entry costs. Finally and unlike in the equilibrium with public signals, profits become nil.

A major lesson comes from our robustness check. The potential harmful effect of free entry on consumer and social welfare occurs only if $\varphi_{r_\sigma} > 0$, i.e. only if bigger firms are assumed to have more to lose when cheating consumers, and no private signals are taken into account. In this case an important policy recommendation can be formulated: when quality is an issue, it is not clear whether authorities should promote competition to increase consumer and social welfare.

In this short paper we tackled the issue of service quality provided by competitive homogeneous firms to fully rational consumers. We initially characterized the first best contract in a static context and then showed that firms have no incentive to provide a positive level of nonobservable quality. Finally, we introduced reputation and showed that the equilibrium quality level is positive. Relying on an equilibrium social convention about the minimum acceptable quality we provided a simple solution to the Stiglitz (1989) objection.

A Appendix

**Lemma 1.** The first best contract is the solution to the problem

$$\max_{q_i, p_i} \{ \alpha q_i - p_i \} \text{ s.t. } \sigma_i [ p_i - c(q_i) ] \geq 0$$

The Lagrangian is $\alpha q_i - p_i + \lambda \sigma_i ( p_i - c(q_i) )$; FOCs are

$$\frac{\partial}{\partial q_i} = 0 \Leftrightarrow \lambda \sigma_i c'(q_i) = \alpha; \quad \frac{\partial}{\partial p_i} = 0 \Leftrightarrow \lambda = \frac{1}{\sigma_i}.$$  

The constraint is hence binding. Substituting $\lambda = 1/\sigma_i$ into $\lambda \sigma_i c'(q_i) = \alpha$ yields the result. S.O.C. wrt $q_i$ is verified: $\frac{\partial^2}{\partial q_i^2} = -c''(q_i) < 0$. To prove that the (1) is the equilibrium contract when the firms compete à la Bertrand and $q$ is observable, it is sufficient to invoke a Bertrand undercutting argument.

**Lemma 2.** The optimal contract is the solution to the problem

$$\max_{p_i} \{ \alpha q - p_i \} \text{ s.t. } \sigma_i [ p_i - c(q) ] \geq 0.$$  

The result follows. To prove that the (2) is the equilibrium contract when the firms compete à la Bertrand and $q$ is nonobservable, it is sufficient to invoke a Bertrand undercutting argument.

**Lemma 3.** Profit $V_{i,t}$ decreases with $c(q^{A}_{i,t})$: the optimal deviation is setting $q^{A}_{i,t} = q$ and, recalling $c(q) = 0$, $V_{i,t}$ rewrites as

$$V_{i,t} \equiv \sigma_i [ p_i - (1 - \tau_{i,t}) c(q_{i,t}) ] + \delta [1 - \varphi(\tau_{i,t}, \sigma_{i,t})] V_{i,t+1}. \quad (9)$$
Since $\partial^2 V_{i,t}/\partial \tau_{i,t}^2 = -\delta V_{i,t+1} \varphi_{\tau} (\tau_{i,t}, \sigma_{i,t}) \leq 0$ under Assumption 1, firm $i$ will not cheat if and only if
\begin{equation}
\frac{\partial V_{i,t}}{\partial \tau_{i,t}} = \sigma_i c (q_i) - \delta \varphi_\tau (\tau_{i,t}, \sigma_{i,t}) V_{i,t+1} \leq 0
\end{equation}
at $\tau = 0$. We assume that our dynamic model is stationary and we then check whether such a solution is admissible. Putting $V_{i,t} = V_{i,t+1}$ with $\tau = 0$ in (9) and omitting subscript $t$ yields
\begin{equation}
V_i = \sigma_i \frac{p_i - c(q_i)}{1 - \delta}.
\end{equation}
Plugging the above value into (10) yields
\begin{equation}
\sigma_i c (q_i) - \delta \sigma_i \frac{p_i - c(q_i)}{1 - \delta} \varphi_\tau (0, \sigma_i) \leq 0.
\end{equation}
Rearranging gives (3).

**Lemma 4.** The problem to be solved is:
\begin{equation}
\max_{\rho, q} \{ \alpha q_i - p_i \} \text{ s.t. } \delta \left( p_i - c(q_i) \right) \varphi_\tau (0, \sigma_i) - (1 - \delta) c(q_i) \geq 0.
\end{equation}
First order conditions with respect to $q_i$ and $p_i$ are:
\begin{align*}
\frac{\partial}{\partial q_i} &= \alpha - \mu \delta \varphi_\tau (0, \sigma_i) c'(q_i) - \mu \left( 1 - \delta \right) c'(q_i) = 0; \\
\frac{\partial}{\partial p_i} &= -1 + \mu \delta \varphi_\tau (0, \sigma_i) = 0,
\end{align*}
where $\mu$ is the Lagrangean multiplier of the constraint, which is binding at the optimum. Substituting the second F.O.C. into the first one and rearranging yields $\alpha = \left( 1 - \delta + \delta \varphi_\tau (0, \sigma_i) \right)/\delta \varphi_\tau (0, \sigma_i)$. Finally, differentiating the Lagrangean with respect to $\sigma$ yields $\varphi_{\tau \sigma} (0, \sigma_i) (p_i - c(q_i)) / \varphi_\tau (0, \sigma_i) > 0$.

**Lemma 5.** Applying the implicit function theorem to the second equation of (4) yields
\begin{equation}
\frac{\partial q_c}{\partial \sigma_i} = - \frac{\alpha \kappa^2 \kappa'}{c''(q_c)},
\end{equation}
hence $\text{sign} (\partial q_c / \partial \sigma_i) = - \text{sign} (\partial \kappa / \partial \sigma_i) = \text{sign} \varphi_{\tau \sigma} (0, \sigma)$, which is positive under Assumption 1.

**Proposition 1.** We solve the game backwards.

(i) Point 5. According to the equilibrium strategy consumers do not buy upon receiving the public signal from firm $i$. Each consumer expects then all the other clients not to buy from firm $i$; its market share is hence anticipated to be very low with the effect that the LHS of (3) tends to 0 given $\varphi_{\tau \sigma} > 0$, while the RHS tends to zero only for $q$ close to $q$. The consumers infer that $q_i \rightarrow q$: this is why they prefer to turn to any other competitor.

(ii) Point 4. We analyze separately the two cases: $n \leq n_{SB}$ and $n > n_{SB}$. In the first case, contract (6) satisfies with the equality the IC constraint (3), hence the consumers accept it since they get the maximum. Indeed, two possible deviations are available to any firm $i$: offering a contract with either (a) better or (b) worse conditions for the clients. In case (a) the IC constraint is either directly or indirectly violated. In the latter situation the mechanism is as follows. Each consumer, according to the equilibrium strategy, expects all the other clients to refuse this offer; firm $i$‘s market share is hence very low. The consumers anticipate that $q_i \rightarrow q$ and prefer to turn to any other competitor. Case (b) is trivial. Consider now the case of $n > n_{SB}$. If all firms offer contract (7), the clients accept it since, given $q = q$, it is the maximum they can
get. Again, two possible deviations are available to any firm $i$, but invoking the same reasoning as above one can show that neither is accepted.

(iii) Points 2 and 3. We first prove that in each period $t \geq 0$ the contract (6) is an equilibrium one for any $n_{SB} \geq n \geq 2$. Such a contract satisfies the IC constraint (3) with equality: if all the firms offer it the consumers accept and the firms get $\Pi_i > 0$ on each contract stipulated at each time $t$. We proved that any other contract would be refused by consumer, hence there is no profitable deviation. With $n_{SB} < n$ the consumers accept only contract (7) which yields zero profits; however any other contract would also bring zero profits, hence there is no strictly profitable deviation.

(iv) Point 1. Suppose first $n < n_{SB}$ firms enter with $q_r (n + 1) > \bar{q}$. Recalling that $q_r$ decreases with $n$, at least an additional firm can enter and offer contract (6) with $n + 1$ instead of $n_{SB}$. Such an offer would be accepted given (8), hence the entrant would end up with positive profits: $n < n_{SB}$ cannot be an equilibrium of the initial entry stage. Now suppose $n > n_{SB}$ firms enter. This implies $q_r (n) < \bar{q}$, hence given (8) no client believes that the entrant and the incumbents will provide $q_i > \bar{q}$. It follows that the firms compete à la Bertrand and offer a contract which entails the maximum utility for the clients with $q_i = \bar{q}$, and zero profits for the firms: entry is not a strictly profitable strategy for an outside firm.

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