In the Multi-Pickup and Delivery Problem with Time Windows (MPDPTW) a set of vehicles must be routed to satisfy a set of client requests between given origins and destinations. A request is composed of several pickups of different items, followed by a single delivery at the client location. This talk introduces two new formulations for the MPDPTW, the 2-index and the asymmetric representatives formulation. In addition, we present an existing 3-index formulation for the MPDPTW and improve it by means of several preprocessing and valid inequalities. We solve the MPDPTW exactly via a branch-and-cut algorithm. We introduce several families of valid inequalities to strengthen the LP relaxations of the proposed formulations. Computational results are reported on different types of instances to firstly highlight the advantage of adding different families of valid inequalities, and then to compare the performance of the different formulations presented. While the heuristic and exact algorithms from the literature prove optimality for 16 instances containing up to 50 nodes, we prove optimality for 41 instances for cases containing up to 100 nodes from the existing benchmark set.