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Giulia Rivolta *(1)*

*(1)* Department of Economics and Management, University of Brescia.

e-mail: giulia.rivolta@unibs.it

**Abstract**

This paper analyses the behaviour of the European Central Bank over the period 1999-2014 through the estimation of monetary policy reaction functions with time-varying coefficients and heteroskedastic error terms. This allows to evaluate whether relevant shifts in the conduct of monetary policy occurred and whether the current financial crisis had an influence on that. The paper considers two different specifications, one with contemporaneous regressors and one with regressors from surveys. The Taylor rule is then enriched with a set of macroeconomic and financial variables with the aim of testing their significance. Results show that the ECB reacts more to forward-looking variables. All the coefficient are found to be stable along the sample so that no shift in the reaction function can be identified and the financial crisis is found to only led to a change in the size of the shock. Finally, we also provide evidence about the fact that the ECB has been actually constrained by the zero lower bound during the recent crisis.

**Keywords**: Euro-area monetary policy, European Central Bank, monetary policy reaction function, time-varying parameters

**JEL Codes**: C11, C54, E52, E58

1 **Introduction**

The beginning of the European Monetary Union (EMU) in 1999 has brought several new issues to the attention of economists. At that time the main interest was to analyse the economic effects of the monetary policy unification in the Euro-area. Then, in subsequent years researchers started to study the conduct of monetary policy by the European Central Bank (ECB henceforth) mainly by estimating Taylor-type monetary policy rules. Our paper aims at contributing to this strand of literature by producing further evidence on the conduct of monetary policy in the Euro area. In particular, this paper estimates monetary policy reaction functions for the ECB with time varying coefficients from 1999 until the end of 2014 with quarterly data.
To better understand the behaviour of the ECB, this paper considers two baseline specifications of the monetary policy reaction function. In both policy rules the monetary authority is assumed to react to annual inflation and output growth but they differ for considering either contemporaneous variables or forecasts so that they will be for simplicity called, respectively, the contemporaneous rule and the forward-looking rule. In the case of the contemporaneous rule, the regressors are not known at the time of the policy decision but with at least one- or two-month lag. Assuming rational expectations, the current variables are instrumented in the first stage of the analysis taking the fitted values from a BVAR model with time-varying coefficient. This allows to avoid the endogeneity problem related to the use of their contemporaneous values and the use of their whole distribution also correct for the generated-regressor bias.

The objective of this analysis is to compare these two different specifications in order to understand which is the best framework for analysing ECB monetary policy decisions. In this sense, our work is closely related to Gorter et al. [2008] but we innovate on the extended dataset and the econometric framework. As regards the sample, the contribution consists in the consideration of the post-2008 period, which will be simply referred to as the “crisis period”. The aim is to evaluate whether relevant changes in the conduct of monetary policy can be detected along the sample and if they can be attributed to the financial-credit crisis and sovereign debt sustainability issues that arose from 2008 on. This assessment is possible allowing for time-varying coefficients and heteroskedasticity.

The reason why we consider time-varying coefficients is that assuming constant coefficients is a too strong restriction since policymakers can react differently to changes in economic variables depending on the actual economic conditions. The importance of allowing for some degree of discretion in the application of a monetary policy rule is stressed, beyond others, by Taylor [1993] that, besides formulating the well-known Taylor rule, critically deal with the practical aspects of taking policy decisions. In particular he pointed out that “(...) in my view, a policy rule need not be a mechanical formula (...). A policy rule can be implemented and operated more informally by policymakers who recognize the general instrument responses that underlie the policy rule, but who also recognize that operating the rule requires judgment and cannot be done by computer”.

The change in policy can take the form of a gradual change or a sudden shift to another regime. These two different views require different model specifications and so it is important to clarify that in this paper time variation is assumed to occur smoothly over time. As a matter of fact, the problem with models considering discrete breaks is that they cannot properly account for gradual policy changes leading to problematic
interpretations when the actual policy changes do not exactly fit the specified model regimes. On the contrary, considering smooth transitions appears to be the most suited approach to deal with monetary policy. From this point of view, our work is also related to Gerlach [2011], Gerlach and Lewis [2014a], and Gerlach and Lewis [2014b] who employed a smooth transition model with heteroskedasticity.

It is important to recognize that a change in behaviour can be due either to a shift in the preferences of the central bank or in the structural economic relations. As shown by Svensson [1997], the coefficients in the monetary policy reaction function are a convolution of the central bank’s preferences and other parameters describing the structure of the economy. Therefore the parameters coming from the estimation of the monetary policy rule will just represent the weight assigned by the central bank to the variables considered and conclusions on the source of their variation would need a further investigation in line with Castelnuovo and Surico [2003] and Assenmacher-Wesche [2006], that we do not pursue here.

The last relevant feature of our model is that also the variance-covariance matrix of the residuals is assumed to have a time-varying component as in Ciccarelli and Rebucci [2006]. Errors’ heteroskedasticity is important to correctly identify parameter time variation as assuming a constant variance-covariance matrix could induce to identify a change in the conduct of monetary policy while in fact what is occurring is just an unaccounted change in the characteristics of the monetary policy shock\(^1\). As a matter of fact, the residuals of the monetary policy reaction function can be interpreted as monetary policy shocks if the so-called recursiveness assumption is valid, i.e. the policy shocks are orthogonal to the regressors and to the other contemporaneous economic disturbances [Christiano et al., 1999]. In this sense, monetary policy could be in principle evaluated also using structural models like vector autoregressions that allows to explore the whole transmission mechanism. However, this type of analysis is based on identification assumptions which are always prone to criticisms. For this reason, we think that a reduced-form approach based on the direct estimation of the policy equation is the best way to evaluate the reaction of a central bank to economic variables. The two-step procedure applied in this paper allows to correctly estimate exogenous monetary policy shocks without necessarily having to identify the entire model structure as the sufficient assumption is that the policy instrument does not influence the given macro variables in the current period.

From an econometric point of view, the model has a state-space representation to which the Kalman filter and smoother can be applied and the estimation is possible through Bayesian simulation techniques. This

\(^1\)Heteroskedasticity is a standard assumption in the Bayesian literature but the present framework differs from those of Cogley and Sargent [2005] and Primiceri [2005] as they assume an autoregressive structure for the error variance to obtain a stochastic volatility model. Our model has the advantage of being parsimonious in the number of parameters to be estimated.
choice is motivated by the fact that allowing time-variation in coefficients greatly increases the number of parameters to estimate bringing about an overfitting problem. Bayesian inference is an efficient solution to this kind of problems because it allows to shrink the dimensionality of the problem by letting parameters come from posterior distributions defined by a narrow set of hyperparameters.

The second stage of our analysis is to extend the monetary policy reaction functions by adding other variables that might have been considered by the ECB. Indeed, one of the purposes of this paper is to test the explanatory power of additional variables itself but also to see whether the inflation and the output growth coefficients are affected. Although not explicitly targeted, these variables may have played a role in the monetary policy decision process as they contain information on future output and inflation.

The last aim of the paper is to examine the issue of the zero lower bound (ZLB henceforth) on interest rate, which is a natural completion when analysing the crisis period. Given our theoretical framework and the estimated reaction function coefficients, it is straightforward to derive the target interest rate and assess whether and when the ECB has been actually constrained.

Results show that the ECB takes into account expected output growth and inflation when setting interest rates. In the forward-looking specification both coefficients on output growth and inflation are positive and greater that one. Instead, in the contemporaneous rule, the inflation coefficient is not significantly different from zero. The inclusion of some additional variables shows that the estimates of the forward-looking rule are much more robust as very few variables are significant and the coefficients of output and inflation remain almost unchanged.

Interestingly, the coefficients of all the policy rules turned out to be constant through time while a spike in heteroskedasticity is detected at the beginning of 2009. This signals that the recent financial crisis did not lead to a shift in the ECB reaction function but just to a change in the size of the shocks. Finally, as regards the ZLB, the ECB have been actually constrained during the crisis.

The structure of the paper is as follows. Section 2 summarizes the monetary policy strategy adopted by the ECB since 1999. Section 3 reviews the literature about monetary policy rules both for what concerns their theoretical specification and the econometric applications with a focus on the Euro area. Section 4 defines the theoretical and econometric specification of the monetary policy reaction function. Section 5 shows the estimation results from different monetary policy reaction functions and section 6 concludes.
2 The ECB Strategy

For the purpose of this work, it is useful to identify which are the macroeconomic variables that are considered by the ECB in its decision process. This section reviews the monetary policy strategy of the ECB in the period 1999-2013 and it is based on information coming from official ECB documents like Monthly Bulletins and speeches of the President, usually after the Governing Council monthly meeting.

The policy strategy of the ECB has been announced in October 1998 and its main objective is the price stability in the Euro-area [ECB, 1998]. The rationale behind this target is that a monetary policy that credibly maintains price stability is assumed to give the best possible contribution to the economic objectives of the European Union by creating an environment in which other policies can be most effective [ECB, 1999]. However, the ECB recognizes that it cannot directly control the price level, but it faces a complex transmission mechanism based on several different channels. This makes it difficult for the ECB to predict the effects of its policy actions as they are also likely to change in response to an evolving economic environment. To address this issue, the monetary-policy strategy is based on two elements: a quantitative definition of price stability and a two-pillar approach for the analysis of the risks to price stability.

Price stability is defined as a year-on-year increase in the Harmonized Index of Consumer Prices (HICP henceforth) of below 2% to be maintained over the medium term [ECB, 1999]. This quantitative value has been chosen to balance the cost of inflation with the necessity for the central bank to maintain an appropriate margin for policy reaction in case of deflationary pressures. On the other hand, the focus on the medium term convey the principle that monetary policy cannot control price developments in the short term with the aim to avoid the introduction of unnecessary volatility into the economy.

The two pillars on which the analysis of price stability is based are the monetary and economic analysis. These approaches are intended to provide two different perspectives on the determination of price developments.

The monetary analysis originally assigned a prominent role to money growth in the assessment of the outlook for price developments taking a reference value for the broad monetary aggregate M3. This strategy has been reviewed in 2003 [ECB, 2003] when the Governing Council decided to no longer consider the reference value of the broad monetary aggregate M3 on an annual basis but to use the monetary analysis as a means of cross-checking the short-term indications coming from the economic analysis from a longer-term point of view.

As regards the economic analysis, its purpose is to provide a broader outlook for price developments by
taking into account the shocks hitting the Euro-area economy and the interplay between supply and demand in the goods and labour markets so that all the risks to price stability can be evaluated. The assessment is made using a wide range of economic indicators that act as leading indicators for prices including wages, the exchange rate, bond prices, the yield curve, measures of real activity, fiscal policy indicators, price and cost indices and business and consumer surveys [ECB, 1999]. Moreover, the economic analysis makes large use of macroeconomic models with the aim of producing projections of the main economic variables, as explained by Issing [2004].

Monetary policy decisions are taken by the Governing Council on the basis of an assessment of the monetary policy stance that can be defined as “the contribution made by monetary policy to economic, financial and monetary developments” [ECB, 2010] while the assessment is the procedure that allows to evaluate whether the effects of monetary policy decisions are consistent with the central bank’s objectives. More in details, the assessment involves two elements: the formation of a view on the medium-term inflation outlook and the identification of the contribution that monetary policy makes to the real economy and the maintenance of price stability. The assessment of the monetary policy stance takes into account a broad range of economic, financial and monetary variables.

As regards economic activity, the main macroeconomic variables considered are: GDP and its components (mainly consumption and investments), unemployment and industrial production. As regards prices: the HICP, inflation expectations, commodity prices, unit labor costs and industrial output prices. The ECB considers also sentiment indicators like consumer and industrial confidence and investors’ sentiment. The most cited financial variables are: the yield curve, market interest rates, nominal and real long-term interest rates, stock market volatility and financial indicators. Of much concern is also the member states’ fiscal position and debt ratios while from 2008 the Euro-area credit conditions gained much attention. Finally, also the developments in the world economy are taken into account with a focus on the US economy.

The severe financial crisis that hit the economy in 2008 has complicated the conduct of monetary policy by the ECB. The financial turmoil started in August 2007 and it initially led to an impaired functioning of money markets. Then, in the first half of 2010, the financial crisis evolved into a sovereign debt crisis. From an operative point of view, the ECB admitted that assessing the monetary policy stance became more difficult as the economic situation was rapidly changing and there was a high degree of uncertainty: the structural economic regularities were not reliable anymore and the monetary policy transmission mechanism was disrupted. In this context the ECB intervened by gradually cutting the key interest rates by 400 basis
points reaching the level of -0.2%, 0.05% and 0.3%, respectively, in September 2014 and by implementing a wide range of non-standard measures aimed at ensuring the necessary liquidity provision to banks and at dealing with tensions on sovereign debt markets. Overall, the ECB response to the financial crisis is stated to be “geared towards the achievement of the ECB’s price stability objective” [ECB, 2010] and all the measures must be considered as temporary.

A broad picture on the conduct of monetary policy in the Euro area is provided by figure 1 which shows the evolution of the three key ECB interest rates and the Euro-area overnight index average rate for the period from January 1999 until December 2014. It is interesting to notice that after the unfolding of the financial crisis the ECB implemented a rapid interest rate cut that culminated in June 2014 when the deposit facility rate became negative and the MRO rate approached the zero. As regard the EONIA rate, it has always fluctuated around the MRO rate but this regularity broke in October 2008. As a consequence of the unconventional measures implemented the overnight money market rate fell significantly below the main refinancing rate towards the deposit facility rate.

3 Literature Review

This section presents a review of the empirical literature about the estimation of monetary policy rules for the Euro-area. In this field the literature follows three main strands: first, there are papers that compare the conduct of monetary policy in different countries before the EMU, then, other works are interested in comparing the Bundesbank with the ECB and, finally, some authors tried to estimate the reaction function
of the ECB.

In the first category falls the seminal paper of Clarida et al. [1998]. They estimate forward-looking Taylor rules for two sets of countries, Germany, Japan and United States and United Kingdom, France and Italy using monthly data over the sample 1979-1994. Results show that the Bundesbank, the Bank of Japan and the Fed responded both to inflationary pressures and output deviations. On the other hand, the Bank of Italy, the Bank of France and the Bank of England responded less aggressively to inflation and they all followed the Bundesbank closely. Finally, the authors calculate in each point in time a “target” interest rate and compare this with the actual interest rate. The interesting findings are that the gap between the actual and the target rate behaves similarly over time for all countries and the central banks started to track the Bundesbank several years prior to the hard ERM (from 1990 to 1992) while with the onset of the hard ERM the gaps between the actual and the target rates widen.

Also Trecroci and Vassalli [2010] estimate reaction functions for five countries (United States, United Kingdom, Germany, France, Italy) over the sample 1971-1998 but they allow for time-varying parameters. They use simple interest rate rules depending on the output gap, inflation expectations and the lagged interest rate. Their findings are that parameters do shift over time in most cases in a smooth and gradual fashion and interest rate policies diverge widely across countries. Most interestingly these differences are evident also across the three Euro-area countries.

Many works focused on the comparison between the conduct of monetary policy in the Euro-area before and after the EMU.

The paper of Hayo and Hofmann [2006] falls in this category. They compare the Bundesbank and the ECB reaction functions specifying a Taylor rule estimated with monthly data and both forward-looking and contemporaneous variables. For the Bundesbank results show that the response to inflation is significantly larger than the response to the output gap. The ECB reaction function has an inflation coefficient that is not statistically different neither to one nor to the Bundesbank’s coefficient. The big difference between the two central bank is in the output reaction coefficient as the one of the ECB is more than twice as large as the one found for the Bundesbank. This discrepancy is proved to be due to the relatively higher interest rate elasticity for the German economy, i.e. a weaker transmission of monetary policy for the Euro-area.

Gerlach and Schnabel [2000] compare the interest rate implied by a Taylor rule with the real one for 13 EMU-area countries over the period 1990-1998 by creating a fictitious central bank and calculating a unique interest rate for the EMU. Results show that the actual interest rate does not differ much from the
one implied by the Taylor rule and the coefficients are robust to the extensions of the monetary policy rule to other variables. The last econometric exercise is to estimate a forward-looking monetary policy rule where regressors are the inflation rate expected into four periods, the output gap and a constant term. Again this specification captures well the evolution of the EMU interest rate.

Sauer and Sturm [2007] estimates several policy reaction functions for the ECB over the sample 1991:01-2003:04 and compare them with the policy rules followed by the Bundesbank. They consider both contemporaneous and forward-looking policy rules. Their results show that the coefficient on contemporaneous inflation is positive but low for the ECB. However this finding is not anymore valid if a forward-looking rule is considered. In this case the inflation coefficient is positive and almost always greater than one. On the other hand the estimates of the output gap coefficient are more stable through different specifications and they are all positive but lower than one. The degree of partial adjustment in the interest rate is found to be significantly large.

Finally, other authors estimate a monetary policy reaction function for the ECB with the aim of studying the conduct of monetary policy in the Euro area.

Gerdesmeier and Roffia [2004] estimate several reaction functions for the Euro-area over the period 1985:01-2002:02 using GMM. In order to do so, they construct measures of aggregate variables and they derive a fictious measure of monetary policy for the period before 1999. The theoretical framework is the one of Clarida et al. [1998] but they also enrich the specification with several other variables and consider different measures of inflation and output. The main results are that both the coefficients of inflation and output are significant and not statistically different from the ones proposed by Taylor [1993] and their magnitude is around 2 and 0.3 respectively. These estimates are not sensitive neither to changes in the measures of inflation and output nor to the inclusion of other explanatory variables.

Carstensen [2006] estimates backward-looking monetary policy reaction functions for the ECB over the first four years of the EMU by employing an ordered probit model for the MRO rate. The main results are that both the inflation and the output gap coefficients are positive but lower than one (0.5 and 0.3 respectively). Further, money growth, money overhang and the real money gap are found to be significant so that it is possible to conclude that the first pillar of the ECB strategy has been important for the policy decisions. Also the significance of the second pillar is tested by considering nominal and real effective exchange rates, an interest rate spread and a real interest rate. Only the interest rate spread is found to have a positive and significant coefficient. Finally also the presence of asymmetries in the policy reaction function is considered.
and only a slight asymmetric effect is found.

Fourçans and Vranceanu [2007] analyse the ECB monetary policy over the period 1999-2006 by means of a qualitative and a quantitative analysis. The estimated monetary policy rule has the lagged short term rate, inflation and an indicator for real activity as regressors. Both contemporaneous and forward-looking rules are estimated. Results show that the ECB responds significantly to future inflation and to different measures of output gap with coefficients smaller than one while the coefficient on contemporaneous inflation is not significant. These findings are similar to those coming from the estimation of a small model of the Euro-area economy made up by the forward-looking monetary policy rule, an IS equation and a Phillips curve.

Blattner and Margaritov [2010] try to find a robust specification of the monetary policy rule of the Euro-area by using a real-time monthly database consisting of 127 series compiled with the data available to each Governing Council meeting over the sample 1999-2007. The first econometric exercise consists of estimating 3300 different specifications of the policy rule and pool the parameter estimates according to some efficiency criteria. The specification of the policy rule is similar to the one in Clarida et al. [2000]. Results show that the ECB is neither purely backward nor forward-looking, but it reacts to a synthesis of the available information on the current and future state of the economy. As regards the magnitude of coefficients, those of the contemporaneous inflation and output are positive but lower than one while those of future inflation and output are often greater than one. In the last part of the paper six factor are extracted from the real-time database and they are used to estimate a policy function. Results are consistent with the previous findings.

A last strand of literature faces the issue of characterising the conduct of monetary policy by the ECB during the crisis.

Gorter et al. [2010] estimate a forward-looking reaction function with both partial adjustment and first-order serially correlated errors over the period 1998-2010 with the aim of analysing the stability of coefficients. They find that the ECB gives priority to price stability and the coefficient of expected inflation is statistically stable over time while the coefficient for expected output gap decreases in the crisis period.

Gerlach and Lewis [2014a], Gerlach [2011] and Gerlach and Lewis [2014b] analyse the interest rate setting behaviour of the ECB by using a smooth transition model that allows for two regimes in the sample period and heteroskedastic error terms. Their main finding is that the ECB reaction function is not stable over time. Gerlach [2011] identifies a shift in the reaction function in mid 2008 while Gerlach and Lewis [2014a] identify a first switch in autumn 2008 and a second switch in late 2010. As regards the coefficient, the ECB
responded more aggressively to expected inflation than to expected output with both coefficients positive and significant in the pre-crisis period and non-significant during the crisis. Moreover the ECB seems to have cut interest rate more rapidly than what the pre-crisis reaction function would have implied. This is compatible with the theoretical literature on optimal monetary policy in the presence of the ZLB which suggests that the central bank should implement an aggressive expansionary monetary policy to maintain long-term interest rate low if it foresees that the ZLB will be binding in the near future\(^2\).

Another recent study that analyse Euro-area monetary policy allowing for time variation in the coefficients is Benchimol [2014]. More in details, he focuses on the role of risk aversion in the Eurozone in a small-scale DSGE model estimated over five (partially overlapping) periods from 1971 to 2011. The findings are that a risk aversion shock increases inflation, decreases output and limit the effect of monetary policy actions. Indeed, the most interesting result regarding monetary policy is that in the last five years of the sample its role in driving output variability has diminished while the role of risk aversion shocks has increased. Further, the nominal interest rate is found to increase its sensitiveness to technology shocks in more recent periods while its sensitivity with respect to risk aversion decreases over time.

Finally, Orphanides [2010] provides an estimate of the short term interest rate as implied by a rule that prescribes to react to expected inflation and GDP growth. This kind of rule, in which the coefficients are both set to be equal to 0.5, turns out to describe well the evolution of the MRO rate in the Euro area suggesting that the reaction function parameters did not change during the crisis.

### 4 The ECB Reaction Function

#### 4.1 Theoretical Specification

Our starting point for the specification of the ECB policy rule is the equation for the target interest rate used by Clarida et al. [1998]:

\[
i_t^* = \bar{i} + \beta \left( \mathbb{E} \left[ \pi_{t+k} | \Omega_t \right] - \pi^* \right) + \gamma \mathbb{E} \left[ x_{t+q} | \Omega_t \right].
\] (1)

Here \( \bar{i} \) is the long-run equilibrium nominal rate, \( \pi_t \) is the rate of inflation between period \( t \) and \( t+k \), \( \pi^* \) is the target level of inflation, \( x_t \) is a measure of output over the period \( t \) and \( t+q \) and \( \Omega_t \) is the information

\(^2\)See for example Reifschneider and Williams [2000], Orphanides and Wieland [2000] and Adam and Billi [2006]. Indeed Gerlach and Lewis [2014a] also produces evidence of the fact that the ZLB has been actually binding from mid 2008 until at least the end of 2009 as the implied target interest rate was negative.
set available to the central bank at time $t$. This equation is a very general representation of a policy rule which can be translated in a contemporaneous or in a forward-looking rule depending on whether $k$ and $q$ are, respectively, zero or positive. Indeed, both these two different types of rules will be estimated and further details are in the next section. Here it is important to notice that, even if the central bank is assumed to target the current value of inflation and output, $\pi_t$ and $x_t$ still need to be taken with expectations because their actual value is not known by the central bank at the time in which it takes its policy decisions.

Few modifications are needed in the last equation.

First of all, the long-run equilibrium nominal rate should be considered as time varying. This nominal interest rate can be defined as the one targeting the natural real interest rate, i.e. the real rate that would have prevailed in an economy with neither nominal rigidities nor cost-push shocks. Barsky et al. [2014] study the role of the natural real rate in a New-Keynesian DSGE model estimated on US data showing that it is highly volatile and procyclical. Therefore, in order to take into account this finding, we will substitute $\bar{\bar{i}}$ with $\bar{\bar{i}}_t$ and define it as follows:

$$\bar{\bar{i}}_t = \bar{\bar{i}} + \pi_t, \quad (2)$$

where $\bar{\bar{i}}_t$ is the natural real interest rate and $\pi_t$ is the inflation rate which is zero only in a fully-flexible economy.

Then, it is also necessary to relax the assumption of a constant inflation target $\pi^*$ as the ECB targets a range of values for inflation, as discussed by ECB [1999] and Kieler [2003], and furthermore Féve et al. [2010] demonstrate that the target might change depending on the economic environment.

Therefore, we assume that the ECB has a target interest rate for the nominal short term interest which depends on the state of the economy as follows:

$$i^*_t = \bar{\bar{i}}_t + \beta (E [\pi_{t,k} | \Omega_t] - \pi^*_t) + \gamma E [x_{t,q} | \Omega_t] \quad (3)$$

As regards the setting of the actual policy rate, two assumptions are made here. First, the actual policy rate is set before the realization of $\pi_t$ and $x_t$ and second, the ECB follows a partial adjustment mechanism for the theoretical reasons explained by Woodford [1999]\(^3\), i.e. the central bank has the tendency to smooth the interest rate and to fix it as a weighted average between the past interest rate and the target, plus a random

\(^3\)Other reasons that can justify the appearance of the lagged interest rate in the monetary policy reaction function are the fact that the central bank operates in an environment of data uncertainty as its decisions are based on real-time data rather than revised ones [Orphanides, 2001] and the existence of serially correlated shocks which are not captured by the empirical rule [Rudebusch, 2002].
shock:

\[ i_t = (1 - \rho) i^*_t + \rho i_{t-1} + v_t. \] (4)

Here \( v_t \) is an exogenous shock to the interest rate which is assumed to be i.i.d.. There can be several economic interpretations for \( v_t \). Christiano et al. [1999] report three different interpretations: \( v_t \) can reflect exogenous shocks to the preference of the monetary authority, it can be due to some technical factors like measurement errors in the preliminary data available leading to an imperfect response of the central bank to changes in the economy or it can derive from the willingness of the ECB to avoid the social costs of disappointing private agents’ expectations so that shocks to their expectations becomes self-fulfilling.

Rudebusch [2002] formulated a different model where interest rate smoothing comes from serially correlated errors. Since then many authors tried to reconcile the high degree of interest rate smoothing found in empirical reaction functions with either the partial-adjustment or the serial-correlation hypothesis, e.g. Coibion and Gorodnichenko [2012] for the United States and Castelnuovo [2007] for the Euro area. We leave this issue open for further research being aware that addressing it would imply the estimation of a more complicated model with a much higher number of parameters.

Substituting equation 3 in 4 we obtain the equation for the actual nominal interest rate:

\[ i_t = \alpha + \psi E[\pi_t|\Omega_t] + \theta E[x_t|\Omega_t] + \rho i_{t-1} + v_t, \] (5)

where: \( \alpha = (1 - \rho) \delta, \psi = (1 - \rho) \beta, \theta = (1 - \rho) \gamma \) and \( \delta = \bar{i}_t - \beta \pi^*_t \).

From this equation it is clear that the parameters coming from the empirical estimation of the monetary policy rule are reduced-form parameters and it will not be possible, in our framework, to pin down both the equilibrium interest rate \( i^*_t \) and the inflation target \( \pi^*_t \) simultaneously. On the other hand, we will focus on commenting the results for the long-run coefficients \( \rho, \beta \) and \( \gamma \).

In the case of the contemporaneous rule, the estimation of equation 5 cannot be directly implemented by substituting to \( E[\pi_t|\Omega_t] \) and \( E[x_t|\Omega_t] \) their contemporaneous values \( \pi_t \) and \( x_t \) as they would be affected by an endogeneity problem. This problem can be overcome by assuming that the ECB has rational expectations and so \( E[\pi_t|\Omega_t] \) and \( E[x_t|\Omega_t] \) are instrumented. Here, instruments will be generally indicated as \( u_t \) and their importance comes from the fact that they allow to get identifying assumptions. So let \( u_t \) be a vector of variables within the central bank’s information set at the time it chooses the interest rate (i.e. \( u_t \in \Omega_t \)) that are orthogonal to \( v_t \). Possible elements of \( u_t \) include any lagged variables that help forecast inflation and
output, as well as any contemporaneous variables that are uncorrelated with the current interest rate shock \( v_t \). Equation 5 and the fact that \( E[v_t|u_t] = 0 \) imply the following set of orthogonality conditions:

\[
E [i_t - \alpha - \psi E[\pi_t, k|\Omega_t] - \theta E[x_t, q|\Omega_t] - \rho i_{t-1}|u_t] = 0.
\] (6)

In this specific case \( u_t \) are the lagged inflation rate and the lagged output measure. These variables will be included as regressors by using the fitted values of a time-varying parameters BVAR model for inflation and output. A more extensive specification of the estimation procedure is postponed to the next sections.

This baseline Taylor-type reaction function is then enriched with several other variables to allow for a clearer identification of the conduct of monetary policy in the Euro-area so that the general specification is the following:

\[
i_t^* = i^* + \beta (E[\pi_t, k|\Omega_t] - \pi^*) + \gamma E[x_t, q|\Omega_t] + \xi E[z_t|\Omega_t].
\] (7)

### 4.2 Data

In this section we take the monetary policy rules to the data.

The monetary policy instrument is assumed to be the short-term money market rate, i.e. the Euro-area overnight index average. This assumption is standard in the empirical literature on the estimation of monetary policy rules\(^4\). Even though the EONIA is not under the direct control of the ECB, because it represents the bank funding conditions on the money market, it closely tracks the key policy rates as it fluctuates around the rate on main refinancing operations and between the deposit facility and the marginal lending facility rates as shown by figure 1. However this relationships broke during the crisis period as the EONIA fell substantially below the repo rate. This happened because of the unconventional monetary policy measures implemented by the ECB that makes the low level of the EONIA a direct expression of policy and does not invalidate its use as monetary policy instrument.

The regressors of the policy rule represent the ECB’s information set. From this point of view we identify two different types of policy rules: contemporaneous and forward-looking. Contemporaneous rules will be estimated with actual data instrumented for correcting for the endogeneity problem. Forward-looking rules will be instead based on survey data. Considering two different baseline specifications will allow us to understand which is the best framework for analysing ECB monetary policy decisions as done by Gorter

\(^4\)See e.g. Bernanke and Blinder [1992], Clarida et al. [1998], Clarida et al. [2000] and Judd and Rudebusch [1998].
As regards the contemporaneous rule, it must be noted that the contemporaneous value of output and of inflation are not available neither to the monetary authority at the time of policy decision nor to markets. For this reason their contemporaneous value can be considered as a measure of expected inflation and output while its lagged value is what it is actually available at the time of policy decisions. In order to make equation 5 estimable, the ECB is assumed to have rational expectations and so the current inflation rate and output gap are instrumented by taking the fitted values of a time-varying parameters BVAR model with one lag of the endogenous variables and of the policy rate. The next section provides the details about the estimation procedure.

The reference series for inflation is the annual growth of the HICP.

A critical aspect for the estimation is the choice of the output variable as in principle several different measures can be used. A large strand of the literature uses the output gap which is meant to capture the difference between the actual and the potential output of the economy and it therefore depends heavily on the estimate of potential output. The problem with the output gap is that there is no formal consensus on how to empirically derive it as different notions of potential output exist. Many studies apply statistical filters like the Horick-Prescott filter, some others use linear and quadratic trends. A further issue is that the empirical estimate of the output gap is strictly dependent on the sample and on the method applied which makes it impossible for a central bank to know the actual output gap in real time. Several papers explored the issue of using imprecise measures of the output gap and its implications in the conduct of monetary policy finding that this is not without consequences. Moreover Gerlach [2007] observes that the output gap never appears in the editorials of the European Central Bank. As a consequence, in this work we will use the approach of Blattner and Margaritov [2010] and estimate growth rules so that \( x_t \) will be annual growth rate of real GDP.

On the other hand, more recent papers use survey data.

As of the forward-looking model, data for inflation and output growth come from surveys and namely we will employ data from the Survey of Professional Forecasters (SPF henceforth) which is published quarterly and reports expectations for the inflation rate, real GDP growth and unemployment in the euro area for

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5. This is the so-called real-time data issue, see e.g. Orphanides [2001]
7. Orphanides et al. [2000], Orphanides [2003a], Orphanides [2003b], McCallum [2001], Ehmann and Smets [2003], and Grigoli et al. [2015]
8. The use of economic growth in reaction functions finds theoretical support in the arguments of Walsh [2003].
9. See e.g. Gorter et al. [2008], Blattner and Margaritov [2010], Gorter et al. [2010], Orphanides [2010], Gerlach [2011], Gerlach and Lewis [2014a] and Gerlach and Lewis [2014b].
several horizons. As in Orphanides [2010] and Gerlach and Lewis [2014a], we will use the forecasts over a
1-year horizon of GDP growth and inflation.

These basic models are then extended to include other variables that might have been taken into account
by the ECB so that both the explanatory power of this additional variables and the robustness of coefficients
on inflation and output are tested. The general rationale of this exercise is that the following variables
potentially contain information about future inflation so that they may have played a role in the interest rate
setting decision, even if they are not explicitly targeted by the ECB.

The additional variables considered are the lagged value of a broad monetary aggregate and of the money
gap, an index of commodity prices, an oil price index, the nominal effective exchange rate, an index of Euro-
area sovereign yield spreads, a stock market price index, and the annual growth rate of bank loans. Further
details over the series used in the estimation can be found in appendix 1. This specification is consistent with
a closed-economy monetary policy rule as there are no foreign inflation and output among the regressors.
The estimation period goes from the first quarter of 1999 to the fourth quarter of 2014 and all the considered
variables are in logarithms except for the EONIA rate.

4.3 Econometric Specification

As we allow coefficients to be time-varying, the ECB reaction function can be re-written in a state-space
form. For each time $t = 1, \ldots, T$, the model has the following structure:

$$y_t = X_t \beta_t + \epsilon_t, \epsilon_t \sim N(0, \sigma \Sigma \epsilon)$$  (8)
$$\beta_t = \beta_{t-1} + \eta_t, \eta_t \sim N(0, \Sigma \eta)$$  (9)

Here $y_t$ is the short-term interest rate, $X_t$ is the $(1 \times n)$ vector of regressors containing the lagged value
of the dependent variable and the $(n-1)$ exogenous regressors, $\beta_t = (\beta_{1t}, \ldots, \beta_{nt})'$ is a $(n \times 1)$ vector of
coefficients and $\epsilon_t$ and $\eta_t$ are the error terms which are normally distributed and orthogonal with each other.

Equation 8 is the measurement equation in which parameters are time-varying. The evolution of parameters
is random, the $\beta$s are treated as latent variables that captures the actual state of the system. In particular,
they follow a random-walk without drift as described by the state equation 9. This assumption is a standard
way of modeling permanent structural changes in behaviour due to fundamental changes in policy regime.

\footnote{Logarithms are applied to the original series and then the necessary transformations are computed.}
see for example Cogley and Sargent [2005] and Primiceri [2005].

As regards variances of the error terms, $\Sigma_\eta$ is an $(n \times n)$ matrix governing the parameters’ evolution, while in the measurement equation errors are heteroskedastic with variance $\sigma_\varepsilon \Sigma_\varepsilon$, i.e. the parameter $\sigma_\varepsilon$ is responsible for time-variation and $\Sigma_\varepsilon$ is a constant scale parameter. Following Ciccarelli and Rebucci [2006], we assume that $\sigma_t$ is distributed as a scaled inverse-$\chi^2$ with $v$ degrees of freedom ($\sigma_t \sim \text{Inv-}\chi^2(\nu, 1)$). This makes the distribution of $\varepsilon_t$ equivalent to a Student-$t$ with $v$ degrees of freedom and scale matrix $\Sigma_\varepsilon$ $(t_v (0, \Sigma_\varepsilon))^{11}$ and so large realizations of the monetary policy shocks are possible. Assuming heteroskedasticity in the residuals is also important to correctly identify parameter time variation. A constant variance-covariance matrix could lead to erroneously identify a change in the conduct of monetary policy while just a change in the characteristics of the monetary policy shock occurred. However this model does not impose unnecessary heteroskedasticity, i.e. if the heteroskedasticity factor is not significant the coefficients’ dynamics and the residuals will be equal to those one would have obtained by estimating an homoskedastic time-varying parameters model. On the other hand, if heteroskedasticity is present, the magnitude of coefficients will be lower than in an homoskedastic framework as some part of the EONIA variations is explained by $\sigma_t$.

With respect of this, we could have in principle adopted a model with stochastic volatility as the one developed by Primiceri [2005] and now widely used in the macroeconomic literature. However this would have substantially increased the number of parameters to be estimated and, given the relatively short sample at our disposal, we preferred to adopt a more parsimonious specification of heteroskedasticity.

Priors are set to be weakly informative. The parameters are assumed to be normally distributed with mean equal to the OLS estimate and variance-covariance matrix equal to four times the OLS variance-covariance matrix. The OLS estimates are calculated on the 3 years previous to the estimation sample for the contemporaneous rules, as usually done in the literature. Instead, forecast data are available only from 1999 on so that, in order to maintain the same estimation sample for the two models, for the forward-looking rules we use the in-sample OLS estimates. The prior over the variance-covariance matrix of the parameters is assumed to be Inverse-Wishart with $\gamma$ degrees of freedom and scale matrix $\Upsilon$. This scale matrix has a very relevant role in determining parameters’ evolution as the greater is the variance, the more time variation will be displayed by parameters. According to what is done in the literature, $\Upsilon$ is considered to be equal to the OLS variance-covariance matrix of the parameters multiplied by the sample size and rescaled by $k^2_{\eta}$ and

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11This can be intuitively understood considering that the Student’s $t$ distribution (with $v$ degrees of freedom) is the distribution of the ratio of two independent random variables: $Z/\sqrt{W/v}$, where $Z \sim N(0, 1)$ and $W \sim \chi^2(v)$. 

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$k_\eta = 0.01$ so that a low degree of parameters time-variation is a-priori assumed\textsuperscript{12}. Finally, the degrees of freedom of the scaled inverse-$\chi^2$ are set to be $v = 5$ as it is the value that maximizes the variance of the prior.

In the model with contemporaneous regressors, output and inflation are made exogenous with respect to the short term interest rate by taking the fitted values from the following time-varying parameters VAR system in structural form:

$$A_t(L) Z_t = D_t R_{t-1} + V_t.$$ \hspace{1cm} (10)

Here $Z_t$ is a $(2 \times 1)$ vector containing output and inflation at time $t$ in natural logarithms, $R_{t-1}$ is the first lag of the monetary policy instrument, $V_t$ is a $(2 \times 1)$ vector of residuals that have zero mean and are serially uncorrelated. Finally, $A_t(L)$ and $D_t$ are the time-varying coefficients matrices with dimension $(2 \times 2)$, and $(2 \times 1)$ respectively and $A_t(L)$ specified in the lag operator $L$ with lag length $p_1 = 1$ \textsuperscript{13}.

The system defined in equation 10 features time-varying coefficients and can be written in state-space form as did for the monetary policy reaction function. The BVAR model is assumed to have the same structure as the monetary policy reaction function, the only thing that differs is the dimensionality.

This two-step estimation procedure is consistent with the VAR models specified by Bernanke and Blinder [1992] and Bernanke and Mihov [1998] but it is also in line with the literature instrumenting the endogenous variables with their lags and applying GMM, see e.g. Clarida et al. [2000]. The policy rate is assumed not to influence the given macro variables contemporaneously and this allows to correctly identify both the parameters of the reaction function and the monetary policy shocks with the reduced-form coefficient and the residuals of model 8-9. The main advantage of this procedure is therefore to correctly identify parameters without having to identify the entire model structure as the equations for inflation and output have a pure statistical specification.

The system in equations 8 and 9 could be estimated with classical methods by obtaining the maximum likelihood estimates of the hyperparameters $\sigma$, $\Sigma_\epsilon$, $\Sigma_\eta$ and then, treating them as true values, derive the estimates of the state variables $\beta_{t:T}$. However, this is not an easy task. Problems can derive from the fact that the likelihood function is defined on a high-dimensional space and that it can have multiple peaks so that the simple maximization does not ensure to find reasonable values for the parameters. Bayesian inference is

\textsuperscript{12}This is in line to what done by, for example, Primiceri [2005] and Stock and Watson [1996] as they want to decrease the degree of time variation in coefficients and improve impulse responses and forecasting analysis. Even though the purpose of this work is different, it is nevertheless important to let time variation be solely data-driven. Appendix 3 test this assumption and show that results are qualitatively confirmed when $k_\eta = 0.1$.

\textsuperscript{13}The system has been estimated also with $p_1 = 2$ but results do not change. Then, for parsimony, the lag length has been chosen to be equal to one.
more efficient as it allows to split the estimation problem in smaller and simpler ones while the use of prior distributions can prevent the maximization algorithm to find implausible maxima.

The Bayesian estimation of the systems of equations 8-9 is possible through the combination of the Kalman filter and the Gibbs sampler as suggested by Carter and Kohn [1994] and Chib and Greenberg [1995]. The procedure is based on the multi-move Gibbs sampler algorithm which iterates the following steps until convergence is achieved: (i) conditional on the model’s hyperparameters and the observed data, generate the entire set of state coefficient $\beta_{1:T}$; (ii) conditional on $\beta_{1:T}$ and the observed data, generate the model’s hyperparameters. The second step of this procedure is straightforward to implement as, conditional on $\beta_{1:T}$, the measurement and the transition equation are two independent regressions. On the other hand, the first step requires the derivation of the distribution of the generic term $\beta_t$ conditional on $\beta_{t+1}$ and the set of observations $y_{1:T}$. Appendix 2 goes into the details of the estimation algorithm. This algorithm generates smoothed estimates, i.e. estimates that are based on the entire set of observations, which are preferable with respect to filtered estimates if, as in this case, the objective is to study the evolution of the latent factors over time.

In order to correct for the generated-regressors bias the two systems are estimated in the same simulation step. For every iteration of the Gibbs sampler, the coefficients of the BVAR system are estimated and used to obtain the fitted variables which are in turns taken as regressors in the second part of the algorithm where the monetary policy reaction function is estimated. By doing so, the generated regressors will change at each iteration depending on the coefficients’ draw meaning that their full distribution is considered in the estimation of the monetary policy reaction function. This allows to take into account the uncertainty connected to the generated regressors and then it is not necessary to correct the error terms as done in Kim [2006] and in Kim and Nelson [2006]. In order to account also for sampling uncertainty, several draws from the posterior distributions are generated for every draw of the regressors.

5 Empirical Results

This section presents the results from different specifications of the monetary policy reaction function. We start presenting some evidence from the estimation of constant-parameters reaction functions. The baseline specifications are Taylor rules with either contemporaneous regressors or forecasts. These policy rules are then extended, in the time-varying framework, to consider other variables that may have played a role in
the interest rate setting decision, namely money growth and money gap, a commodity price index and oil prices, the nominal effective exchange rate, a spread index, a stock market index and bank loans. This exercise follows a marginal approach as the variables are added one at a time so to evaluate their marginal significance and whether the coefficients of the baseline specifications are affected. Further details about the data used can be found in appendix 1.

All the figures with the results summarize the posterior distributions, i.e. the median and the 16th and 84th quantile of the distributions.

For all the following models the algorithm generates 40000 draws from the marginal distributions and the first 10000 draws are used as burn-in and so they are discarded. In the case of the contemporaneous rule, as regressors comes from the estimation of a BVAR model, it is necessary to account for sampling uncertainty. This is accomplished by generating 10 draws from the posterior distribution of parameters for every draw of the regressors so that in this case the Gibbs sampler generates 400000 draws.

To eliminate autocorrelation in some parameters, posterior distributions are built retaining only one draw every five cycles of the Gibbs sampler algorithm, i.e. by using 60000 draws from the remaining 300000 in the case of contemporaneous rules and 6000 draws from the remaining 30000 in case of forward-looking rules. Convergence is checked for every model by using both graphical analysis and convergence diagnostics. Appendix 4 shows some convergence diagnostics for the baseline contemporaneous and forward-looking model.

5.1 Constant-Parameters Reaction Functions

In this section we estimate the baseline models, i.e. the ones with only output growth and inflation among regressors, assuming constant parameters and homoskedastic error terms. This will give us a basis for comparison with the results in the following sections.

For sake of comparability with the results in the following sections the estimation is based on Bayesian inference but, given that all the sources of time-variability have been eliminated, the reaction functions become standard regressions with Normal - Inverse-Gamma priors. In the case of the contemporaneous rule, the regressors are, as in the time-varying framework, fitted values from a Bayesian-VAR model for GDP growth and inflation.

Table I shows the mean of the coefficients and their 16th and 84th quantile in brackets. The coefficients refer to the specification in equations 3 and 4 so that $\alpha$ is the constant, $\rho$ is the autoregressive coefficient,
and $\beta$ and $\gamma$ are the long-run coefficients of, respectively, inflation and output growth.

Following the results of the contemporaneous specification, the ECB strongly reacted to output growth while the inflation coefficient is not significant. On the contrary, the specification with forecast data highlights that the response to expected inflation has been very strong. Here also the output growth coefficient is higher than in the contemporaneous rule. These results will be confirmed in the time-varying framework so that we will better discuss their implications in the next sections.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Contemporaneous Rule</th>
<th>Forward-Looking Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.062 [-0.171; 0.046]</td>
<td>-1.051 [-1.197; -0.906]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.901 [0.874; 0.929]</td>
<td>0.833 [0.815; 0.851]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.358 [0.972; 1.687]</td>
<td>1.875 [1.681; 2.067]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.185 [-0.383; 0.805]</td>
<td>3.12 [2.672; 3.741]</td>
</tr>
</tbody>
</table>

### 5.2 Contemporaneous Taylor Rule

This specification has time-varying coefficients and considers output growth and inflation as regressors. Figure 2 shows the results.

The first thing to notice is that the coefficients remained almost constant through time. This is in line with Orphanides (2010) but at odds with Gerlach (2011), Gerlach and Lewis (2014a) and Gerlach and Lewis (2014b), as they identify a shift in the reaction function coefficients at the end of 2008. However the heteroskedasticity factor displays a relatively low peak in 2001 and a higher one at the end of 2008 meaning that a change in the size of the shock occurred. This result is confirmed by all the models estimated and highlights that allowing for heteroskedasticity is important for capturing the correct dynamics of the coefficients. On the contrary, our result find theoretical and empirical support in Benchimol and Fourçans (2012) as they find that the level of relative risk aversion in the economy does not influence the parameters of the Taylor rule.

As regards the coefficients of the reaction function, figure 2 displays $\alpha$ and $\rho$ of equation 5 and $\gamma$ and
from equation 3, which have been derived combining the reduced-form coefficients $\theta$ and $\psi$ (neither displayed nor commented here) with $\rho$. The same is valid for all the figures in the following sections. The long-run coefficients show that the ECB responded aggressively to output growth with a mean coefficient that is always close to one and slightly increasing through time. This result can be reconciled with an inflation targeting behaviour by considering the informative power of output growth in predicting future inflation. On the contrary, the inflation coefficient is never significant. In light of the findings regarding output, this result should not be surprising as it may simply signal that the ECB is much more concerned with future inflation rather than current one.

This results are qualitatively similar to Fourçans and Vranceanu [2007] and Blattner and Margaritov [2010] but at odds with Castelnuovo [2007] and Gerdesmeier and Roffia [2004] as in both work the coefficients on inflation and output are found to be, respectively, close to 2 and to 0.3.

Finally, in line with much of the literature, there is also evidence of a high degree of partial adjustment.

One problem connected with our results is that they violate the Taylor principle which states that the nominal interest rate should rise by more than the increase in the inflation rate in the long run in order to have an increase in the real interest rate. However Bullard and Mitra [2007] demonstrate that policy inertia has a crucial role in defining determinacy, existence and learnability of rational expectation equilibria. Indeed, a high degree of policy inertia increases the region of determinacy and induce learnability and this gives credit also to policy rules that do not respect the Taylor principle.
Figure 2: Contemporaneous Taylor Rule

Parameters of the Rule

Residuals and Heteroskedasticity Factor

5.3 Forward-Looking Taylor Rule

This specification uses as regressors the forecasts for inflation and output growth over a 1-year horizon coming from the Survey of Professional Forecasters. Figure 3 shows the results.

As before, there is no evidence of time variation in coefficients but instead the crisis led to an increase in the size of the shock from mid 2008 to mid 2009 which is however smaller with respect to the contemporaneous rule. Coming to the coefficients of the policy rule, it appears that the ECB strongly reacted to both GDP growth and inflation. The coefficient of GDP growth is higher but not significantly different from the one of the contemporaneous rule while the coefficient of inflation is now comprised between 2.6 and 3.6 suggesting that the ECB pursued a stabilizing policy towards inflation as prescribed by the Taylor principle. This find-
ing are qualitatively consistent with Fourçans and Vranceanu [2007], Sauer and Sturm [2007], Castelnuovo [2007], Blattner and Margaritov [2010] and Gorter et al. [2008] as they all compare the outcomes of Taylor rules with backward-looking variables with those coming from rules with expectations data. However, our coefficient on expected inflation is much higher than the one estimated in the literature. On the other hand, our results are at odds with Gerlach and Lewis [2014a] as they find a much lower sensitivity to expected output growth and inflation in the pre-crisis period and negative and non-significant coefficients during the crisis.

Figure 3: Forward-Looking Taylor Rule
Parameters of the Rule

Residuals and Heteroskedasticity Factor
5.4 Extensions

In this section both the contemporaneous and the forward-looking Taylor rules are extended for taking into account other variables that might have influenced the ECB in setting the interest rate. In fact, given the two-pillar strategy for the analysis of price stability discussed in section 2, many variables can potentially enter the evaluation of the monetary policy stance and monetary policy decision process.

It is important to remark that these variables are not explicitly targeted by the ECB but are directly or indirectly mentioned in the official statements signalling that they all may convey some information on the economic outlook and price developments. Therefore, it becomes interesting to statistically verify if and to what extent other variables are significant for policy decisions. To this purpose, we follow a marginal approach, i.e. the variables are added one by one so that also their effect on the coefficients of output and inflation can be more clearly evaluated.

5.4.1 M3

The first variable considered is money. As explained in section 2, monetary analysis had a preeminent role in evaluating price stability up to 2003 and afterwards its role has been revised.

Recently Benchimol and Fourçans [2012] studied the role of money and risk aversion in the transmission of shocks in the Eurozone finding that money plays a minor role when risk aversion is low while it explain a significant part of output fluctuations when risk aversion is high. The authors also analyse a Taylor rule where different measures of money are considered. Interestingly, the only measure of money that turns out to be significant is money gap. However many other authors augmented a Taylor rule with a money variable which is usually the annual growth of M3. Therefore, in order to be consistent with the literature, here we test both money growth and money gap as explanatory variables\(^\text{14}\). We use the first lag of both variables to avoid endogeneity issues.

Figures 4 and 5 show that the inclusion of M3 growth into the monetary policy reaction function does not lead to any change in the coefficients of the baseline Taylor rules. The M3 coefficient is significant only in the contemporaneous rule from 2006 up to the end of 2008. Consistently, also the residuals and the heteroskedasticity factors are unchanged. This finding is consistent with, among the recent studies, Gorter et al. [2008] and Gerlach and Lewis [2014b].

\(^{14}\)For money growth we use the annual percentage change in M3 while the money gap is computed as the difference between the current value of M3 and its trend coming from the Hodrick-Prescott filter with \(\lambda = 1600\).
As regards the money gap, in line with Benchimol and Fourçans [2012], figures 6 and 7 show that it is significant in the forward-looking policy rule and its inclusion leads to a decrease in the coefficient of expected inflation which is of the same magnitude as the coefficient of money. To the best of our knowledge, only Carstensen [2006] found a similar result while Castelnuovo [2007] considered a money gap variable that turned out to be non-significant.

Figure 4: Contemporaneous Taylor Rule with Money Growth

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
Figure 5: Forward-Looking Taylor Rule with Money Growth
Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
Figure 6: Contemporaneous Taylor Rule with Money Gap
Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
5.4.2 Commodity and Oil Prices

Commodity and oil prices are the inputs of the production process and, for this reason, their prices represent a primary source of inflation in the economy. Here we consider the first lag of their annual change to evaluate whether the ECB has a different sensibility with respect to commodity inflation and to final price inflation.

Results are displayed in figures 8, 9, 10 and 11. Commodity prices are not significant in the contemporaneous rule, in line with Gerdesmeier and Roffia [2004], while they have a negative but very small coefficient in the forward-looking rule, but this does not have a meaningful economic interpretation. Oil prices are significant in the contemporaneous rule and their inclusion lead to a decrease in the coefficient of inflation that is probably due to their relevance in forecasting future inflation. This interpretation finds support in the
results of the forward-looking Taylor rule as there is no reaction to oil prices.

Figure 8: Contemporaneous Taylor Rule with Commodity Prices
Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
Figure 9: Forward-Looking Taylor Rule with Commodity Prices
Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
Figure 10: Contemporaneous Taylor Rule with Oil Price
Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
5.4.3 Exchange Rate

In this specification, the annual change in the nominal effective exchange rate\(^{15}\) is added. The rationale for considering this variable is that import prices directly affect inflation so that the exchange rate contains information on future prices and, more in general, it conveys the effects of international developments. Many authors evaluated its role but results are not homogeneous.

Figures 12 and 13 display the results. The exchange rate coefficient is significant in both rules and shows that when the Euro depreciates the ECB increases interest rates to curb inflation. The magnitude of the coefficient is greater in the contemporaneous rule and, as a consequence, the inflation coefficient is now

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\(^{15}\)The exchange rate is defined such that when it increases the Euro currency appreciates.
almost significant. This is consistent with Gerlach [2007] and Gerlach and Lewis [2014b] (they find it is significant only in the pre-crisis period), but not in line with Gerdesmeier and Roffia [2004] and Carstensen [2006] as in their backward-looking policy rules no measure of exchange rate is significant. Overall these results confirm that the exchange rate is seen as having a predictive power on prices even when inflation is measured with expectation data. This is at odds with Castelnuovo [2007] and Gorter et al. [2008] who found that the exchange rate is not relevant in, respectively, forward-looking rules and both rule with ex-post or with expectations data.

Figure 12: Contemporaneous Taylor Rule with Nominal Effective Exchange Rate

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
5.4.4 Bond Yields Spreads

The recent financial crisis also involved public finances of several European countries leading to tensions in sovereign debt markets. For this reason an index of bond yield spreads is added to the monetary policy reaction function. The index is computed as a weighted sum of government bond yields’ spreads, with respect to Germany, of ten Euro-area countries and weights are given by the relative debt-to-GDP ratios as calculated by the Eurostat. To avoid endogeneity the first lag of the index is considered. Results are displayed in figures 14 and 15.

In the contemporaneous rule the coefficient of the new variable is always negative and becomes significant from 2009 on. The coefficients of the other regressors are unaffected. Overall this reflect the fact that
during the sovereign debt crisis the yield spreads became relevant in capturing the state of the economy so that it is not surprising that the ECB tried to curb tensions on sovereign debt markets.

These findings are not confirmed in the forward-looking policy rule as, in that case, the spread index turns out to have a positive and always significant coefficient. The output and inflation coefficients are now higher than before and by the same amount of the size of the spread coefficient. This however does not have a meaningful economic interpretation in light of the crisis developments.

In the literature, the only other paper testing a risk spread variable is Gorter et al. [2008] finding that it is not significant, neither in the backward-looking nor in the forward-looking policy rule and this is only partially in line with our findings.

Figure 14: Contemporaneous Taylor Rule with Spread Index

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
5.4.5 Stock Market Volatility

This specification adds a stock market index to the baseline Taylor rule. The appropriateness of including measures of financial market volatility has been widely discussed in the theoretical literature on monetary policy and regained attention in light of the recent financial crisis, see e.g. Bernanke and Gertler [2000], Cecchetti et al. [2000] and Kuttner [2012]. Overall the literature finds scant support for targeting asset prices as an aggressive inflation-targeting rule can substantially stabilize the economy even in case of asset bubbles [Bernanke and Gertler, 2001] and Orphanides [2010] reviews the motivations and implications of an activist and a non-activist strategy. One of the motivations that can lead a central bank to consider movements in the stock market is pursuing financial stability. We do not want to enter here the debate on whether
financial stability considerations should shape monetary policy but rather empirically evaluate whether the ECB pursued other objectives besides output and inflation stabilization.

The index considered here is the Dow-Jones Euro Stoxx 50. Its annual variations are included into the reaction function and, in order to escape any endogeneity issue, the first lag is considered. Results are in figures 16 and 17.

In line with Gerdesmeier and Roffia [2004], the Euro Stoxx index has a positive and significant coefficient in both rules but its inclusion in the contemporaneous rule led to a decrease in the output growth coefficient and an increase in the inflation coefficient. In the forward-looking rule the additional coefficient is much lower and the sensitivity to output and inflation is unchanged. These findings highlight that the stock market has been considered to have some forecasting ability over output growth and that the ECB pursued the objective of financial stabilization, mainly during the crisis period.

Figure 16: Contemporaneous Taylor Rule with Stock Prices
Parameters of the Policy Rule

![Graphs showing parameters of the policy rule with stock prices.](image)

Residuals and Heteroskedasticity Factor

![Graphs showing residuals and heteroskedasticity factor.](image)
5.4.6 Credit to Private Sector

In this specification the monetary policy reaction function is enriched with a credit variable, namely the amount of loans to non-financial corporations, to households and to other financial institutions coming from the statistics on the monetary financial institution (MFI) sector. The rationale for adding this variable into the policy rule is that the credit channel plays a prominent role in the monetary policy transmission process [Bernanke and Gertler, 1995]. Therefore, it is natural to assume that a central bank may monitor also developments in the credit market to evaluate the monetary policy stance. These series are available from 2003 and their annual growth rate is considered. In order not to lose any data, priors are calibrated on in-sample OLS values for both rules.
Results are displayed in figures 18 and 19 and they show that the bank loans are significant in explaining the behaviour of the ECB only when using contemporaneous output and inflation. In the case of the rule with expectation data, loans are not significant but their inclusion still leads to a small decrease in the coefficient of output growth.

Finally, the residuals and the heteroskedasticity factors of both models are comparable with those from the previous specifications.

Figure 18: Contemporaneous Taylor Rule with Bank Loans

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
5.5 The Target Interest Rate

In the context of the recent crisis and economic development the issue of the existence of a zero lower bound has become central in the monetary debate. Therefore, in order to evaluate whether the zero lower bound has been actually binding for the ECB, in this section we focus on the analysis of the target interest rate implied by the policy rules previous estimated. The target interest rate is defined by equation 3 and represents the desired interest rate of the monetary authority and only depends on the state of the economy.

Figure 20 displays the target interest rate of the baseline contemporaneous policy rule while figure 21 displays the one implied by the forward-looking rule. Every chart plots the distribution of the target interest rate (the black dashed lines), i.e. the median and the 16th and 84th quantile, against the actual EONIA rate.
(the blue solid line).

The first thing to notice is that the two different specifications lead to very similar results as regards the dynamics of the target interest rate and this counts as a robustness check for our results.

Turning to the analysis of the results, in both the rules the target interest rate is estimated to be positive (or at least non-negative at the end of both 2001 and 2003) up to the beginning of the recent financial crisis at the end of 2008. Then it declines sharply and remains significantly negative up to mid 2010 (in the contemporaneous rule) and up to the beginning of 2010 (in the forward-looking rule) signalling that the zero-lower bound was binding. Interestingly, the last part of the sample is the one in which the target rate differs the most across the two types of rules. The contemporaneous rule finds that the ECB has been constrained from mid 2011 until the end of 2013 that represents the period in which the consequences of the Euro-area sovereign debt crisis have been most severe, while the forward-looking rule finds that the target rate remains negative from mid 2011 on.

Finally, it is important to highlight that these results are almost identical to those obtained by Gerlach and Lewis [2014a] that propose the same exercise but based on a different framework. They use monthly data and estimate a smooth transition model based on a reaction function as in Judd and Rudebusch [1998] over the sample 1999-2011. It is striking to notice that both the dynamics and the point estimates of the target rate are remarkably similar for most of our policy rules. This of course gives support to our results.

Figure 20: Target Interest Rate vs EONIA Rate for the Baseline Contemporaneous Policy Rule
6 Conclusions

This paper produced some evidence on the conduct of monetary policy in the Euro area over the period 1999-2014 by using time-varying coefficient reaction functions with heteroskedastic errors estimated with Bayesian techniques.

Dealing with data of the 2008-2014 period is not an easy task as most of the macroeconomic variables show huge variations which can easily invalidate any econometric analysis. A model with time-varying coefficients combined with heteroskedastic errors offers a very flexible framework that can ideally adapt and capture changes in the macroeconomic environment.

The paper considered both contemporaneous and forward-looking reaction functions whose baseline specifications feature annual inflation and output growth. For the contemporaneous rule, in order to avoid endogeneity issues, inflation and output have been instrumented by using a time-varying parameters BVAR model. On the other hand, the regressors of the forward-looking model come from the survey of professional forecasters of the ECB.

As in much of the recent literature, forward-looking variables are found to have a larger weight in the policy rule with respect to contemporaneous variables. In both specifications the coefficients on output growth is positive but its is around 1 in the contemporaneous rule and in the range of 1,5-2 in the forward-looking rule. Instead, only expected inflation is significant and with a coefficient close to 3 implying a very
strong reaction in line with the Taylor principle.

As regards time variation, we do not find support for the findings of Gerlach [2011], Gerlach and Lewis [2014a], and Gerlach and Lewis [2014b] as the parameters of our reaction functions remain almost constant through the sample even when we a-priori impose a higher degree of time variation. We only identify a significant change in the size of the shock which can be attributed to the financial crisis. This of course does not invalidate the results of the previous papers but just highlights that the choice of the model plays a significant role in driving the results. One possible explanation for this strong model-dependent results may be that the sample at our disposal is still too short for identifying results that are robust through different specifications and model choices. Further research in this field is definitely needed and maybe a broader historical perspective will help in clarifying the behaviour and the role of the ECB.

These baseline specifications are then extended adding further variables that might have been taken into consideration by the ECB in setting the interest rate, namely money growth and money gap, a commodity price index, an oil price index, the nominal effective exchange rate, a government bond yield spread index, a stock market index, and bank loans. The variables are added one at a time so that their marginal influence on the inflation and output coefficients can be evaluated. This exercise shows that the estimates of the forward-looking rule are much more robust as very few variables are significant, i.e. money gap, the nominal effective exchange rate and stock prices, and the coefficients of output and inflation remain almost unchanged. On the other hand, in the contemporaneous rule almost all the additional variables turn out to be significant. Overall this seems to be due to the lack of forward-looking perspective rather than to a true relevance of the variables considered.

As a last point, we computed the implied interest rate target and we find that the the ZLB has actually constrained the ECB during the crisis.

Acknowledgements

A preliminary version of this paper was part of my PhD thesis and it would have not been possible for me to start working on it without the support of Roberto Casarin, to whom I am very grateful. The paper has also largely benefited from the comments and suggestions I received from Alessandro Missale, Luca Fanelli, Efrem Castelnuovo, Emanuele Bacchiocchi, Fabio Verona, Matteo Ciccarelli, Gianni Amisano, Michele Lenza and Luca Dedola. I also thank two anonymous referees for the valuable comments.
References


APPENDIX 1. Data Description

The monetary policy reaction functions are all estimated over the period 1999-2014. For the contemporaneous rules, the overall data sample goes from the first quarter of 1996 to the last quarter of 2014. The first three years of observations are used to calculate the OLS values necessary to initialize the Kalman filter. In this case two of the regressors comes from the estimation of a VAR model starting in January 1996 and, for lack of data, the OLS values to initialize the Kalman filter are calculated in-sample. This procedure is not necessary for the forward-looking rule as the baseline regressors are forecasts. As in this case the series start in 1999, the OLS values are calculated in-sample so that no data are lost.

Variables are quarterly and the logarithmic transformation is applied, except for the EONIA which is taken in level. The EONIA rate is constructed by the ECB as the average of the daily EONIA rate calculated by the European Banking Federation.

Variables used as regressors are:

- the real GDP (in annual growth rate): the series is constructed taking into account the evolving membership of the Euro area and it comes from Datastream;
- HICP: the series come from the ECB;
- expected annual real GDP growth: 1-year ahead forecast from the Survey of Professional Forecasters;
- expected annual inflation: 1-year ahead forecast from the Survey of Professional Forecasters;
- M3: the series come from Datastream;
- the Thomson Reuters/Jefferies CRB Index: it is a commodity futures price index and it is comprised of 19 commodities sorted into 4 groups with different weightings (petroleum based products, liquid assets, highly liquid assets, diverse commodities)\(^\text{16}\) and it comes from Datastream;
- the oil price index: it is the price of Brent crude oil 1-month Forward contracts (free on board) per barrel and data come from Datastream;
- the nominal effective exchange rate: it considers 12 trading partners, it is adjusted using the CPI and it is defined such that an increase indicates an appreciation of the Euro currency, the series comes from Datastream;

\(^{16}\)For further details see: http://thomsonreuters.com/products_services/financial/thomson_reuters_indices/indices/commodity_indices/
• a sovereign bond yield spread index: it is calculated as a weighted sum of government bond yields’ spreads (all taken from Datastream) with respect to Germany of ten Euro-area countries (Austria, Belgium, Finland, France, Greece, Ireland, Italy, Netherlands, Portugal and Spain) and weights are given by the relative debt-to-GDP ratios calculated by the Eurostat;

• the Euro Stoxx 50 index: it is a stock price index including 50 of the largest and most liquid stocks (blue-chips) of the Eurozone, the series comes from Datastream;

• bank loans in the Euro area to non-financial corporations, to households and to other financial institutions: these data come from the statistics on the monetary financial institution (MFI) sector provided by the ECB\footnote{Further information can be found in the Manual of MFI Balance Sheet Statistics available at: http://www.ecb.europa.eu/pub/pdf/other/manualmfbalancesheetstatistics201204en.pdf}.

\section*{Appendix 2. Estimation Procedure}

\subsection*{A Priors and posteriors\footnote{Definitions and derivations in this section follow Gelman et al. [2003].}}

This section presents the derivation of the posterior distributions for the monetary policy reaction function. The same structure has been used for the BVAR model.

The complete-data likelihood of the model is:

\begin{equation}
L(y|X_t, \beta, \Sigma_{\epsilon}, \sigma, \Sigma_{\eta}) = \left(\prod_{t=1}^{T} (2\pi \sigma_t^2)^{-\frac{1}{2}} |\Sigma_{\epsilon}|^{-\frac{T}{2}} \exp \left\{ -\frac{1}{2} \sum_{t=1}^{T} (y_t - X_t \beta_t)' (\sigma_t \Sigma_{\epsilon})^{-1} (y_t - X_t \beta_t) \right\} \cdot (2\pi)^{-\frac{T}{2}} |\Sigma_{\eta}|^{-\frac{T}{2}} \exp \left\{ -\frac{1}{2} \sum_{t=1}^{T} (\beta_t - \beta_{t-1})' \Sigma_{\eta}^{-1} (\beta_t - \beta_{t-1}) \right\} \right) \end{equation}

where $\beta = (\beta_0, \beta_1, \ldots, \beta_T)$ and $\sigma = (\sigma_1, \ldots, \sigma_T)$.

To implement the Gibbs sampler it is necessary to derive the conditional posterior distributions from the product of the likelihood and the priors.

The parameters’ priors are assumed to be independent with each other so that the joint prior is:

\[ p(\beta, \Sigma_{\epsilon}, \sigma, \Sigma_{\eta}) = p(\beta) p(\Sigma_{\epsilon}) p(\sigma) p(\Sigma_{\eta}) \]
For the slope coefficients $\beta$, a time-varying Minnesota prior is assumed, i.e. the coefficients follow a random walk (see Litterman [1986] for details), where errors are assumed to be normally distributed with zero mean and variance-covariance matrix $\Sigma$. In details, the prior on the coefficients is the following:

$$\beta \sim N \left( \hat{\beta}_{OLS}, 4 \cdot \text{Var} \left( \hat{\beta}_{OLS} \right) \right)$$ (12)

For the constant scale parameter of the error term $\Sigma_e$ a diffuse prior is assumed:

$$p(\Sigma_e) \propto |\Sigma_e|^{-1}$$ (13)

This corresponds to the Jeffreys prior density $|\Sigma_e|^{-(k+1)/2}$ with $k = 1$ (see Jeffreys [1961] for details). The Jeffreys prior density is the limit of an inverse-Wishart distribution, which is a conjugate prior distribution for the covariance matrix of the multivariate normal distribution, with 1 degree of freedom and scale matrix that tends to zero.

The time-varying component of the variance of the error term is assumed to be distributed as a scaled inverse-$\chi^2$ distribution with $\nu$ degrees of freedom and scale parameter 1:

$$\sigma_t \sim \text{Inv} - \chi^2(\nu, 1)$$ (14)

The degrees of freedom are set as $\nu = 5$ as it is the value that ensures the maximum depart from normality for $\epsilon_t$. In fact, $\epsilon_t | \sigma_t \sim t_\nu(0, \Sigma_e)$ converges in distribution to $N(0, \Sigma_e)$ as $\nu$ approaches infinity because the mean of $\sigma_t$ tends to one and its variance tends to zero in the limit. Moreover for $\nu \leq 4$ the variance of the distribution is infinite and so $\nu = 5$ is the value that maximizes the prior variance but restricting it to be finite.

The variance-covariance matrix of the parameters is assumed to be distributed as an inverse-Wishart with $\gamma$ degrees of freedom and scale parameter $\Upsilon$:

$$\Sigma_\eta \sim IW(\gamma, \Upsilon)$$ (15)

Here $\gamma = T_{OLS}$ and $\Upsilon = k_\eta^2 \cdot T_{OLS} \cdot \text{Var} \left( \hat{\beta}_{OLS} \right)$. This means that the degrees of freedom are imposed to be equal to the sample size over which OLS are calculated while the scale matrix is assumed to be equal to the OLS variance-covariance matrix of the parameters estimated on the pre-sample and it is multiplied by the sample size. In order to make the prior uninformative, this quantity is rescaled by a factor $k_\eta = 0.01$. This
set-up does not strictly impose time variation but lets it be determined solely by data.

Posteriors distributions for the parameters are derived from the product between the likelihood and the relative prior as they are assumed to be independent.

In particular, the full conditional posterior distribution of the constant part of the residuals’ variance $\Sigma_\varepsilon$ is an inverse-Wishart distribution with $T$ degrees of freedom and scale matrix $S^{-1}$:

$$p(\Sigma_\varepsilon | y, X, \beta, \sigma, \Sigma_\eta) \propto |\Sigma_\varepsilon|^{-(T+1+1)/2} \exp\left\{ -\frac{1}{2} \text{tr} \left( S \Sigma_\varepsilon^{-1} \right) \right\} = IW \left( T, S^{-1} \right),$$  

(16)

where: $S = \left[ \Sigma_t (y_t - X_t \beta_t) (y_t - X_t \beta_t)' \right]$.

As we are in the univariate case ($\Sigma_\varepsilon$ is a scalar), the inverse-Wishart distribution degenerates into an inverse-gamma distribution with shape parameter $\alpha = \frac{T}{2}$ and scale parameter $\beta = \frac{S}{2}$.

The full conditional posterior distribution of $\sigma_t$ is a scaled inverse-$\chi^2$ distribution with $v + 1$ degrees of freedom and scale matrix $s_t^2$:

$$p(\sigma_t | y, X, \beta, \Sigma_\varepsilon, \Sigma_\eta) \propto \sigma_t^{-(v+1)/2} \exp\left\{ -\frac{(v+1)s_t^2}{2\sigma_t} \right\} = Inv - \chi^2 \left( v + 1, s_t^2 \right),$$  

(17)

where: $s_t^2 = \left[ \frac{v}{v+1} + \frac{(y_t - X_t \beta_t) (y_t - X_t \beta_t)'}{v+1} \right]$.

In general, to obtain a draw $\theta$ from an $Inv - \chi^2 \left( v, s_\theta^2 \right)$ distribution it is necessary to draw $x$ from a $\chi^2$ distribution with $v$ degrees of freedom and then let $\theta = \frac{vs_\theta^2}{x}$.

The full conditional posterior distribution of $\Sigma_\eta$ is an an inverse-Wishart distribution with $\bar{\gamma}$ degrees of freedom and scale matrix $\bar{\Upsilon}$:

$$p(\Sigma_\eta | y, X, \beta, \Sigma_\varepsilon, \sigma) \propto |\Sigma_\eta|^{-(\bar{\gamma}+1)/2} \cdot \exp\left\{ -\frac{1}{2} \text{tr} \left( \bar{\Upsilon} \Sigma_\eta^{-1} \right) \right\} = IW \left( \bar{\gamma}, \bar{\Upsilon}^{-1} \right)$$  

(18)

where: $\bar{\gamma} = \gamma + T$ and $\bar{\Upsilon} = \left[ \bar{\Upsilon} + \Sigma_t (\beta_t - \beta_{t-1}) (\beta_t - \beta_{t-1})' \right]$.

The derivation of the joint conditional posterior distribution of the state variables is analysed in the next section.
B  Procedure to estimate the state variables

The procedure to estimate the time-varying coefficient models is based on Carter and Kohn [1994] and Chib and Greenberg [1995]. Here $\beta$ indicates a general vector of latent factors, $y_{1:T}$ is the set of observations and $\varphi$ is the vector of hyper-parameters.

The posterior distribution of parameters is simulated from the multi-move Gibbs sampler which generates simultaneously all the state vectors form the joint distribution $p(\beta_{0:T}|y_{1:T}, \varphi)$ using analytical filtering and smoothing relations, as proposed by Carter and Kohn [1994]. For this purpose the Kalman filter and smoother can be applied. Therefore the objective is to simulate the sequence of parameters vectors $\{\beta_t\}$ given the whole set of observations of the dependent variable $y_{1:T}$ and the remaining parameters $\varphi$. The procedure is based in the simulation of the state vectors sampling from $p(\beta|y_{1:T}, \psi)$ in reverse time order by means of the recursive factorization of the smoothing density. The steps of the algorithm are the following:

- $\beta_T \sim p(\beta_T|y_{1:T}, \psi) = N(\hat{\beta}_T, P_T)$
- $\beta_{T-1} \sim p(\beta_{T-1}|\beta_T, y_{1:T-1}, \psi) = N(\hat{\beta}_{T-1}, P_{T-1})$
- ...
- $\beta_t \sim p(\beta_t|\beta_{t+1}, y_{1:t}, \psi) = N(\hat{\beta}_t, P_t)$
- ...
- $\beta_1 \sim p(\beta_1|\beta_2, y_1, \psi) = N(\hat{\beta}_1, P_1)$

Here the Gibbs sampler to obtain the marginal posterior distributions of the parameters runs iteratively through the conditional distributions 16, 17, 18 and the previous steps for 40000 times and the first 10000 iterations are discarded. Then, to correct for autocorrelation in draws, only one draw each five is retained. so that posterior distributions are made up of 6000 draws which becomes 60000 for contemporaneous rules as, in order to account for sampling uncertainty related to generated regressors, the Gibbs sample generates 10 draws for each draw of the regressors. The only thing needed to initialize the procedure are the initial states of the variables which are taken from the OLS estimation of the monetary policy reaction function over the period 1996 Q1 - 1998 Q4 for the contemporaneous rule and in sample for the forward-looking rule.
APPENDIX 3. Allowing for More Time Variation

A crucial aspect of using Bayesian techniques in the choice of priors and, when dealing with time-varying parameters, the scale factor $k_\eta$ in the prior on the coefficients’ variance covariance matrix plays a key role. The role of this constant is to rescale the scale matrix of the Inverse-Wishart prior in order to make it less-informative so that if time-variation is detected in the coefficients it will only be due to the likelihood function, i.e. time-variation should come from data and not being artificially imposed. The choice of the value of $k_\eta$ can also be driven by empirical reasons used as metrics of the goodness of the model like improving forecasting accuracy or avoiding misbehaviours.

In our application we are primarily driven by the willingness of being consistent with the literature and let data be the only source of time variation so that we followed Primiceri [2005] and set $k_\eta = 0.01$. However, other less restrictive values can be used. The effect of having a bigger scale matrix in the prior for the coefficients’ variance-covariance matrix would be to better fit data through a higher degree of time variation with the drawback that this might only be driven by the prior.

In order to evaluate the effects of a tighter prior in our context, the figures 22 and 23 show the results for the two baseline models estimated with $k_\eta = 0.1$. For both policy rules the residuals and the heteroskedasticity factors do not change much with respect to the case with $k_\eta = 0.01$. On the other hand, coefficients display a higher degree of time variation as expected. However, the qualitative interpretation of the result does not change much.

In the contemporaneous rule the coefficient of output growth is almost stable and slightly lower than before even though it cannot be considered statistically different. Instead, the inflation coefficient is found to display some more time variation, mostly at the beginning of the sample where it is even bigger than one implying that the ECB would have satisfied the Taylor principle. In the rest of the sample the coefficient is positive only from 2006 to mid 2008 and it is not different from zero in the remaining periods.

The coefficients of the forward-looking policy rule are even less time varying and the only thing to notice is that the GDP growth coefficients is declining through time.

Overall, even allowing for a higher degree of time variation through priors, it seems that the reaction function coefficients remained almost constant in our sample giving support to the results of the previous sections.
Figure 22: Contemporaneous Taylor Rule with $k_\eta = 0.1$

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
Figure 23: Forward-Looking Taylor Rule with $k_\eta = 0.1$

Parameters of the Policy Rule

Residuals and Heteroskedasticity Factor
APPENDIX 4. Convergence Analysis

This appendix evaluates the convergence of the MCMC algorithm by means of the Matlab code for CODA containing convergence diagnostics modeled after S-Plus CODA. The function computes diagnostics based on Raftery and Lewis [1992] and Geweke [1992]. Here we decided to report only some of the available statistics and namely those that have been thought to be most relevant to gauge whether the algorithm has converged.

As many models have been estimated in this paper, it has been decided to report convergence statistics for the two baseline models, i.e. the rules with output growth and inflation.

Figure 24 report the autocorrelation function (at lag one, five and ten) of the full chain of the Gibbs sampler (with the burn-in period and before thinning) and of the posterior distribution of parameters for both models. For the contemporaneous Taylor rule the first 256 values represents the autocorrelation of the four monetary policy rule coefficients, the values from 257 to 320 are the autocorrelation of the heteroskedasticity factor and the last 17 values are, respectively, the autocorrelation of the time-invariant part of the residuals’ variance and those of the 16 elements of the parameters’ variance-covariance matrix. Instead, for the forward-looking Taylor rule, the partition of autocorrelations is different; the first 208 values are the autocorrelation of the coefficients and the values from 209 to 260 are the autocorrelation of the heteroskedasticity factor while the last 17 values are the same as before.

The aim here is to justify the use of the thinning procedure. For the full chain the autocorrelation at the first lag is quite high especially for the covariance matrix of the reaction function coefficients. This problem is solved when applying thinning and retaining only one draw every five. On the other hand the dependence across draws decays very rapidly as at the fifth lag it is always below 0.1.

Table II summarizes the Raftery and Lewis [1992] diagnostics. Overall the required number of runs is always far below the total number of iterations of the Gibbs sampler and the diagnostics highlight an improvement in the convergence properties of the chain when thinning is applied.

Finally, as in Primiceri [2005], we consider the relative numerical accuracy (RNE) of Geweke [1992] for the posterior distribution of parameters and to summarize the information into the inefficiency factor. The RNE is the ratio between the variance of an iid sequence of draws from the posterior distribution and the variance of the Gibbs sampler. Here it has been computed considering a 4% tapered window for the

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19 This code has been written by J.P. LeSage and can be freely downloaded from the website www.spatial-econometrics.com.
20 These diagnostics has been computed for the quantile 0.025 and the remaining parameters are set at standard values of 0.01 and 0.95 for, respectively, the desired accuracy and the required probability of attaining the required accuracy.
estimation of the spectral density at frequency zero. The inefficiency factor is the inverse of the RNE and it is an estimator of the k-th autocorrelation of the chain. Table III summarizes the distribution of the inefficiency factor across the different group of parameters and shows that the inefficiency factor is always far below 20, as prescribed.

To conclude, the convergence diagnostics are fully satisfactory.
Table II: Raftery and Lewis (1992) Diagnostics

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Contemporary Rule, Posterior Distribution

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Forward-Looking Rule, Full Chain

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Forward-Looking Rule, Posterior Distribution

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Table III: Inefficiency Factor

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<th>Min</th>
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