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Applications of Conditional Value-at-Risk Beyond Finance: A Literature Review

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Applications of Conditional Value-at-Risk Beyond Finance: A Literature Review

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Abstract

The increasing complexity and uncertainty of the current economic system implies that many problems involve decisions under uncertainty and, hence, with unknown outcomes. Despite the control of risk has been a major issue in financial optimization since the seminal work by Markowitz in 1952, only in the last decades decision makers operating in other application domains became aware of the importance of analyzing and controlling the risk. In financial applications, shortfall or quantile risk measures are receiving an ever increasing attention. The Conditional Value-at-Risk (CVaR) is one of such measures. This survey provides an overview of the main contributions from the literature where the CVaR is incorporated into an optimization approach applied to a context different from finance. We have classified the papers according to their application areas, which range from classical topics in operations research, such as supply chain management, scheduling, and networks, to other subjects like those related to the energy sector and medicine. For each area, we provide an overview of the main problems studied. Finally, we indicate some open research directions.

Keywords: Literature Review, Conditional Value-at-Risk, Optimization under Uncertainty, Logistics, Energy, Medicine.

1 Introduction

The world is increasingly complex and uncertain, causing a perception of risk in decision makers operating in a variety of application domains, and generating a growing request for mechanisms devoted to control or limit the risk involved. This especially affects applications in the broad context of supply chain management Heckmann et al. [2015], but also concerns areas such as engineering and medicine. As a result, the risk-aversion of a decision maker has become an important additional criterion while making decisions, even in areas different from financial optimization. To stress on this point, we quote from Eppen et al. [1989] that, while addressing a problem faced by General Motors (GM), claim:

> *"In any decision under risk, expected profit is not the only objective. Management is also concerned about the risk involved. This is especially true for a set of decisions like the one faced by GM, where large amounts of money and the careers of many individuals are involved."*

In financial applications, the importance of considering risk is largely acknowledged and well-elaborated, and many different measures have been proposed. In 1952, Markowitz [1952] first formalized the problem of determining the optimal investment in a portfolio of financial assets as a bi-criteria optimization problem, where the mean return is maximized and a risk measure is minimized. In this seminal work, the variance was used to measure the risk, and a quadratic optimization model was devised to control the trade-off between risk and return. The paper by Markowitz has posed the fundamental basis for the development of a large part of the modern theory of financial optimization. Although variance is suitable for the case where the outcome distribution is close to be symmetric, it is not appropriate in the general case when distributions may be asymmetric. Since the paper by Markowitz, several other risk measures have been considered in financial applications. Whereas some of these measures, such as the mean-absolute deviation, may be viewed as an approximation of the variance, shortfall or quantile-based risk measures have rapidly gained wide popularity during the first decade of the 21st century. One of such measures is the *Conditional Value-at-Risk*, henceforth called CVaR, first developed in Rockafellar and Uryasev [2000]. CVaR is strongly related to another risk measure called Value-at-Risk, or VaR for short, which is heavily used in various financial and engineering problems, including military, nuclear, and airspace applications (Sarykalin et al. [2008]). Intuitively speaking, the VaR of a portfolio of assets, given a specified probability level α, can be defined as the smallest threshold value η such that the probability that the loss on the portfolio exceeds η is α. The value α is chosen by the decision maker, and is often called confidence level. As pointed out by Rockafellar and Uryasev [2002], a very serious shortcoming of VaR is that it does not provide any indication about the severity of losses beyond its value. Indeed, Sarykalin et al. [2008] highlight that one can significantly increase the largest loss exceeding the VaR, but the VaR risk measure will not change. The CVaR overcomes this limit affecting the VaR, as it measures the conditional expectation of losses above η (see Rockafellar and Uryasev [2000]). In other words, the CVaR approximately (or exactly under certain conditions) equals the average of some percentage of the worst-case loss scenarios.

There are several reasons explaining the success of CVaR as a risk measure. From a practical point of view, CVaR penalizes only the negative deviations with respect to an efficiency target (downside risk measure); it is sensitive to extremely negative outcomes but it is not as conservative as a Minimax approach. From a theoretical viewpoint, CVaR is a coherent risk measure, i.e., it guarantees the consistency with intuitions about rational risk-averse decision makers. From a computational point of view, CVaR optimization can often be embedded in an optimization problem by adding linear constraints and continuous variables, i.e., without increasing the expected complexity of the resulting optimization model. Therefore, it is not surprising that in recent years several authors incorporate CVaR as an additional criterion in their optimization problems, even while facing a number of applications different from optimization in finance.

Aims and Scope of the survey. The literature where CVaR is employed in financial and related applications is vast and ever increasing. Over the last decade, the number of authors who have studied the use of CVaR in application areas different from quantitative finance is substantially increasing, motivating the need for structuring and organizing the related literature. In Figure 1 we show the temporal evolution of the references considered in this survey.

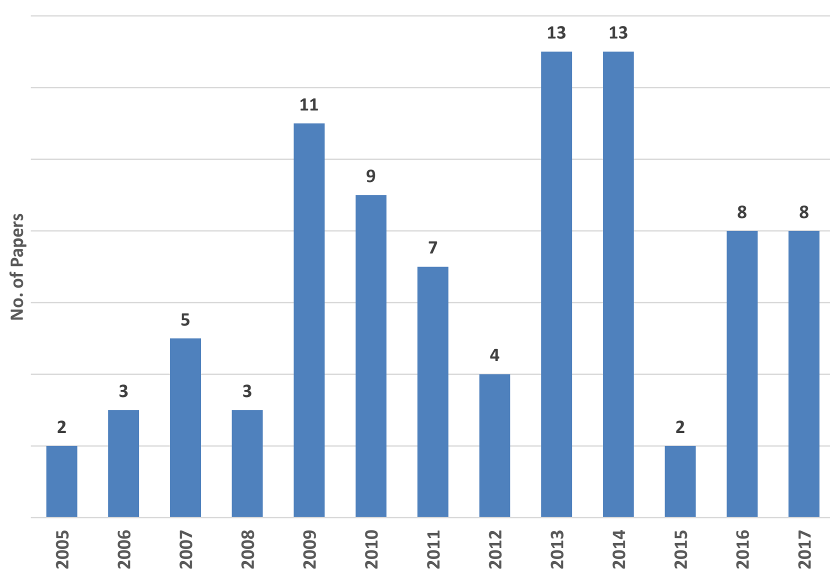

Figure 1: The evolution over time of the papers reviewed.

This survey aims at providing a classification and systematic overview of the foremost contributions in the literature where the CVaR is applied in a context different from quantitative finance. The scope of this paper is to cover those articles where CVaR is employed in an optimization phase to support the decision maker. Hence, it does not include papers where CVaR is computed only as a post-optimization statistic to validate or compare different solutions. CVaR has also been used in deterministic contexts. Even though it is often called with a different name, CVaR and other closely related measures have been applied to optimization problems where concern is given to a fair and equitable distribution of resources among several entities. Along this line, some researchers have studied location-allocation problems and allocation problems related to communication networks. The recent survey by Ogryczak et al. [2014] provides an overview of this class of problems. To the sake of brevity, in the following we mention only those articles that incorporate CVaR in a deterministic context and that are not covered by Ogryczak et al. [2014].

Classification used. The classification of the papers reviewed in this survey is depicted in Figure 2. In the following, we will first present those articles that deal with a classical topic in the Operations Research and Management Science (OR/MS) literature. These topics are Inventory management, Supply chain management, Transportation and traffic control, Location and supply chain network design, Networks, and Scheduling. Altogether they represent roughly 70% of the articles surveyed. A different topic is Energy, which covers issues specifically related to the supply chain in the energy sector. Medicine concerns a specific application of radiation therapy, where CVaR is applied in a deterministic context. Finally, the label Other is referred to a number of different applications, that do not fit in any the above categories.

The survey is organized in sections corresponding to the above categories. Within each section, we cluster the papers with similar characteristics into groups. Some papers could be included in more than one area or group. In these cases, we included them in the group that we believe best describes the application/problem under consideration.

We have collected and reviewed only papers that have been published in international and peer-reviewed scientific journals. In order to achieve an acceptable level of quality of the references included herewith, we have not considered articles appeared in conference proceedings and electronic journals.

Structure of the paper. The structure of the paper is as follows. In Section 2, we briefly recall the main definitions and properties concerning CVaR as a risk measure. Then, we review the papers collected by application area. As already stated, we follow the classification provided in Figure 2 and mentioned above. We begin in Section 3 by providing an overview of the literature related to inventory management problems.

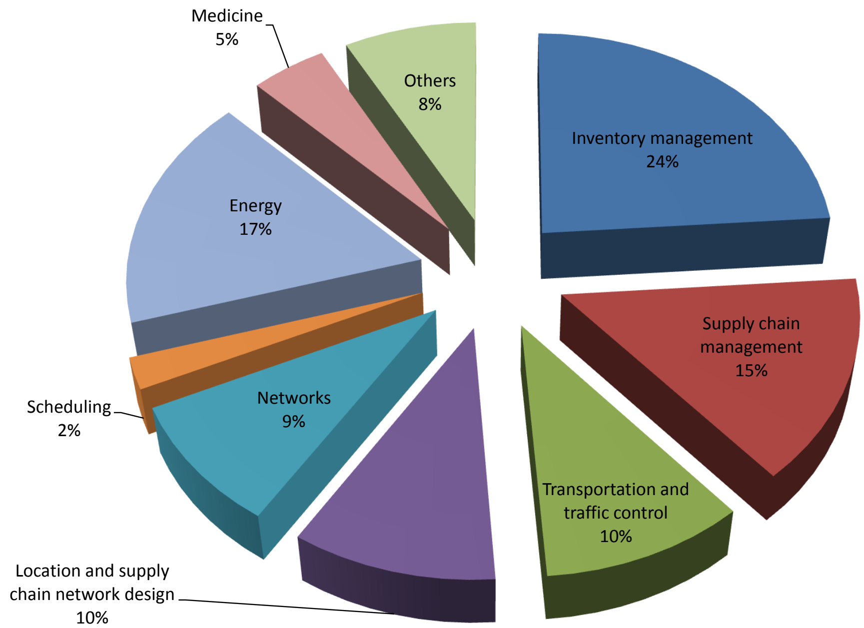

Figure 2: The classification per area of the literature reviewed.

In Section 4, we survey the literature where CVaR has been applied to supply chain management problems, whereas in Section 5 we survey the literature on transportation and traffic control. Section 6 is devoted to location and supply network design applications, while Section 7 is devoted to network problems. This part addressing classical topics in OR/MS finishes with Section 8 collecting two references on scheduling problems. Section 9 surveys the literature addressing applications in the energy sector, Section 10 considers medicine applications, and, finally, Section 11 collects the literature related to additional applications that do not fit in any of the above categories. A conclusive discussion where we identify possible future research directions is given in Section 12.

2 Conditional Value-at-Risk: An overview

In the following, we provide an introductory description of the CVaR, along with its basic properties, using the notation from Sarykalin et al. [2008]. It is beyond the scope of this survey to provide a detailed description of this measure of risk. Hence, we refer the reader interested in further details on the CVaR to the paper by Rockafellar and Uryasev [2000], the follow-up articles by Pflug [2000], Krokhmal et al. [2002], Rockafellar and Uryasev [2002], and the tutorial by Sarykalin et al. [2008].

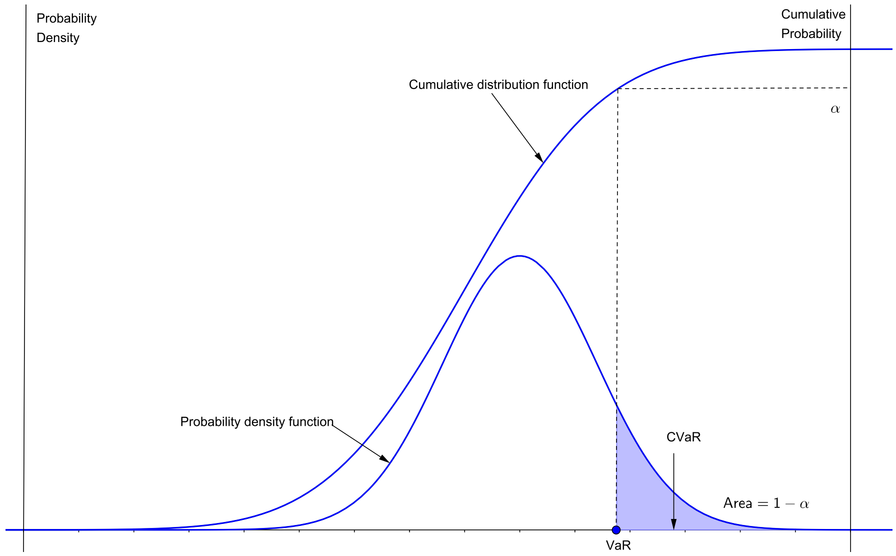

Figure 3: Illustration of the concepts of VaR and CVaR for a continuous loss function.

Let X be a random variable with cumulative distribution function $F_X(z) = \mathbb{P}\{X \leq z\}$. Note that X may have the meaning of loss or gain. Following most of the related literature, we assume in the following definitions that X represents a loss. For a given confidence level $\alpha \in (0,1)$, the VaR is the α-quantile, i.e.,

$$\text{VaR}_\alpha(X) = \min\{z|F_X(z) \geq \alpha\}.$$

For random variables with continuous distribution functions, CVaR equals the conditional expectation of X, given that $X \geq \text{VaR}_\alpha(X)$, i.e.,

$$\text{CVaR}_\alpha(X) = \mathbb{E}(X|X \geq \text{VaR}_\alpha(X)).$$

Therefore, given the same confidence level α, the VaR is a lower bound for CVaR (see Sarykalin et al. [2008]). The above definition is the basis for the name Conditional Value-at-Risk (Sarykalin et al. [2008]). Figure 3 provides an intuitive representation of the concepts of VaR and CVaR for a continuous loss function. The definition of CVaR for general distributions (i.e., with a possibly discontinuous distribution function) is more involved as one may need to split a so-called probability atom. For a formal definition of the CVaR for general distributions, we refer the reader to Rockafellar and Uryasev [2002]. Given the goal of this survey, it is however worth recalling that for discrete random variables, i.e., when probabilities can be represented by using scenarios rather than densities, CVaR has the relevant advantage that it can be optimized by means of Linear Programming (LP) methods.

Furthermore, defining the $\text{CVaR}_\alpha(X)$ for all confidence levels α in $(0,1)$, completely specify the distribution of X (Sarykalin et al. [2008]). Indeed, a strong risk-aversion is captured by large values of α, that is, values close to 1. In such situations, the VaR will be large and the CVaR will depend only on a small fraction of possible outcomes of X, namely the largest losses (refer to Figure 3 for an intuitive explanation). In these cases, the CVaR is strictly close to a classical Minimax approach. On the opposite, for values of α close to 0, the VaR will be small and the CVaR based on a large fraction of possible outcomes of X. In these latter cases, the CVaR tends to be equal to the expected value of X. In other terms, it is possible to capture different degrees of risk-aversion by simply changing the value of the quantile parameter α. Typical values of α used in financial applications are 0.9, 0.95, and 0.99.

Artzner et al. [1999] propose an axiomatic approach to risk measurement defining a *coherent risk measure* as a function that satisfies four properties: monotonicity, sub-additivity, positive homogeneity, and translational invariance. All these are simple, mathematical properties that guarantee the consistency with intuitions about rational risk-averse decision makers. For instance, the sub-additivity property, which some popular measures (including the VaR) do not possess, requires that adding together two risky assets cannot increase the value taken by the measure of risk, i.e., it favors diversification. Pflug [2000], among others, prove that the CVaR is coherent, whereas Ogryczak and Ruszczyński [2002] show that it is consistent with the paradigm of second degree stochastic dominance. The following additional arguments speak in favor of the CVaR. From a mathematical viewpoint, CVaR optimization can be reduced to convex programs, and, as mentioned above, to linear programs for discrete distributions (Sarykalin et al. [2008]). Additionally, symmetric risk measures, like the variance, may be inadequate when applied to asymmetric distributions because they equally penalize desirable (i.e., overperformance) and undesirable (i.e., uderperformance) deviations. In this respect, the CVaR is superior to these risk measures as, for appropriate values of α, it penalizes only a fraction of the undesirable deviations.

3 Inventory management

This section provides a description of the research studying inventory management problems where the CVaR is used as a measure of risk. Almost all the papers reviewed in this section address a variant of the classical newsvendor problem. In its basic form, it is an unconstrained and single-period problem, characterized by one perishable (or seasonable) product with stochastic demand, in which the newsvendor must place a single order before the demand is realized. The *newsvendor problem* calls for determining the optimal quantity of a single order that must be placed before observing the demand when there are shortage and surplus costs. Traditional models assume that the newsvendor's objective is to maximize the expected profit or to minimize the expected total cost. We refer the reader interested in further details on the newsvendor problem to the survey by Qin et al. [2011]. Note that, in principle, the newsvendor problem can be interpreted as the problem faced by the manager of a simplified supply chain which involves only one decision maker, and, therefore, could have been included in Section 4. Nonetheless, we prefer to clearly separate the papers on the newsvendor problem from those reviewed in the latter section, which have a broader and more structured view of the supply chain.

A wide variety of newsvendor problems that employ the CVaR have been studied in the literature. Most of these problems are variants obtained modifying one single characteristic of the above basic framework, and each variant is often addressed in few (or even only one) paper. Given the simplicity of the basic problem and the variants studied, most of the authors are able to derive analytically closed-form solutions to the problem at hand. A summary of the main characteristics of each paper is reported in Table I. Column *Obj.* shows the papers where the objective function of the problem aims at optimizing only the CVaR, whereas column *Weigh. Obj.* denotes those problems where the CVaR is combined in the objective function with some other functions. Column *Cons.* indicates those problems where the CVaR appears in the constraints. Finally, column *Add. Details* reports some additional characteristics of the problem studied, along with some information on the solution approach adopted.

General risk-averse newsvendors. Gotoh and Takano [2007] are, to the best of our knowledge, the first authors to extend the classical newsvendor problem to incorporate risk-aversion by using the CVaR. They show that, under some assumptions on the probability distribution of the demand, closed-form solutions can be derived for this problem when the CVaR is applied to the profit and, separately, the total cost function. For the case of the multi-product and constrained newsvendor problem, they propose two scenario-based LP optimization models assuming that the demand distribution is discrete. The two formulations mainly differ in the objective function: one maximizes the weighted difference between the expected profit and the CVaR, whereas the other minimizes the CVaR imposing a lower bound on the expected profit. Computational experiments are reported for randomly generated test problems where the LP models are solved with XPRESS. Katariya et al. [2014] provide a thorough comparison of the optimal order quantities of a risk-neutral and three risk-averse newsvendors using the CVaR, the mean-variance, and the expected utility maximization,

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Gotoh and Takano [2007]	√	√		SProd: CFS; MProd: LP
Katariya et al. [2014]	√			CFS: Using results from Gotoh and Takano [2007]
Arıkan and Fichtinger [2017]		√		Spectral risk measures; CFS
Sun et al. [2013]	√			LP
Xinsheng et al. [2015]	√	√		CFS
Jammernegg and Kischka [2007]	√			RA&RS; CFS
Jammernegg and Kischka [2013]	√			RA&RS; Constrained model
Jammernegg and Kischka [2009]	√			UDD; RA&RS; CFS
Ahmed et al. [2007]	√	√		UDD; Single-period and MPer
Qiu et al. [2014]	√	√		UDD; Second-order cone and convex programs
Zhou et al. [2008]			√	MProd; LP
Borgonovo and Peccati [2009]	√			MProd; Specific profit function
Zhang et al. [2009]			√	MPer; Sample average approximation
Chen et al. [2009]	√			Demand price-dependent; CFS
Xu [2010]	√			Demand price-dependent; Emergency orders; CFS
Xu and Li [2010]	√	√		Variable shortage costs; CFS
Wu et al. [2013b]	√			Uncertain shortage costs; CFS
Chahar and Taaffe [2009]	√	√	√	Customer selection; MILP
Abdel-Aal and Selim [2017]	√			Market selection; MProd; Non-linear models
Wu et al. [2013a]	√	√		Uncertain procurement; CFS
Wu et al. [2014]	√			Competitive newsvendors; CFS

[a] SProd: Single-Product; CFS: Closed-Form Solutions; MProd: Multi-Product; LP: Linear Programming
RA&RS: Risk-Averse and Risk-Seeking; UDD: Uncertain Demand Distribution
MPer: Multi-Period; MILP: Mixed-Integer LP

Table I: References in inventory management problems: A summary of the main characteristics.

respectively, as decision criteria. The optimal solutions for the newsvendor using the CVaR are based on the results provided by Gotoh and Takano [2007]. Arıkan and Fichtinger [2017] study a risk-averse newsvendor problem using general *spectral risk measures*. The latter class of risk measures has been introduced in Acerbi [2002] as a sub-class of coherent risk measures, which encompass the CVaR as a special case. Arıkan and Fichtinger [2017] reformulate some of the results obtained for general spectral measures to the case of a newsvendor that uses the CVaR. Sun et al. [2013] consider a newsvendor that uses the CVaR to control the occurrence of extreme losses. The problem is cast as an LP model that minimizes the CVaR subject to a constraint on the minimum expected profit. The authors determine efficient frontiers of the expected profit against both the CVaR and the optimal order quantities solving the LP model by means of MATLAB. Xinsheng et al. [2015] consider a newsvendor that aims at minimizing the expected losses given by the sum of shortage and surplus costs. They derive analytically closed-form solutions for this problem, both considering a risk-neutral newsvendor and two risk-averse decision makers. Regarding the latter two newsvendors, risk-aversion is captured using the CVaR of the expected losses. The difference among them is that one aims at minimizing the CVaR, whereas the other minimizes a weighted function of both the expected losses and the CVaR.

Risk-averse and risk-seeking newsvendors. Jammernegg and Kischka [2007] use the CVaR to incorporate into a classical newsvendor problem the risk preferences of both risk-averse and risk-seeking decision makers. To this aim, they consider as objective function a linear combination of two CVaR functions: one computing the worst-case (i.e., the smallest) profit realizations, and the other contemplating the best-case (i.e., the largest) profit realizations. They derive, under certain conditions, closed-form solutions for the optimal order quantity. In a later article, Jammernegg and Kischka [2013] extend the previous research, proposing a constrained optimization model that optimizes the objective function proposed in Jammernegg and Kischka [2007] subject to a limit on two performance criteria, measuring the level of service provided and the probability of losses, respectively. Subsequently, they analyze the same problem under the assumption that the decision maker is not able to specify explicitly the risk preferences. In this case, they use the prescribed performance measures to derive those risk attitudes.

Uncertain demand distributions. An important assumption in the classical newsvendor problem is that the decision maker has complete knowledge of the demand probability distribution. Jammernegg and Kischka [2009] extend the research reported in Jammernegg and Kischka [2007] to a situation where the above assumption does not hold, and the only information available is that the underlying demand distribution belongs to a set of potential distributions that can be ordered using first- and second-order stochastic dominance rules. Ahmed et al. [2007] also investigate risk-averse inventory problems where the underlying demand distribution is not known with certainty, and derive analytically optimal policy structures. Their study focuses mainly on coherent risk measures, and includes the CVaR as a special case. After examining the optimal solution structure of a single-period risk-averse newsvendor problem, the authors extend their analysis to a multi-period setting, and, finally, to a multi-period problem where the ordering cost includes a fixed cost. Along a similar line of research, Qiu et al. [2014] analyze a risk-averse newsvendor problem when the demand distribution belongs to ellipsoid or box uncertainty sets. They propose two alternative objective functions: the first maximizes the CVaR as a function of the worst profits, whereas the second maximizes a linear combination of expected profits and CVaR. For both objective functions, and using standard robust optimization techniques, they cast the problem as a second-order cone or a general convex program, depending on which of the above-mentioned uncertainty sets is considered. Some numerical results are reported which focus on comparing, for each of the objective functions proposed, the solutions obtained changing the uncertainty set used and the level of risk-aversion of the decision maker.

Multi-product risk-averse inventory models. Zhou et al. [2008] study a multi-product risk-averse newsvendor problem where product demands are independent random variables, each one described by a known distribution function. The problem calls for the determination of the optimal order policy of each product, subject to upper and lower bounds on each quantity ordered, and a limit on the total capital spent procuring the products. Zhou et al. [2008] cast the above problem as an LP optimization model, with a limit on the CVaR of the worst cost realizations. Computational experiments are conducted solving the proposed optimization model with MATLAB. In contrast with the classical newsvendor setting, Borgonovo and Peccati [2009] formulate the profit function as the total revenue minus the sum of a fixed order cost and the holding costs paid for the products held in stock. Borgonovo and Peccati [2009] study this problem comparing the optimal decisions of a risk-neutral decision maker to those of three risk-averse newsvendors adopting the variance, the mean-absolute deviation, and the CVaR, respectively, as risk measures. Numerical experiments are conducted under some assumptions on the demand distribution. The results focus on comparing the optimal decisions of the four decision makers studied, and highlight that introducing risk-aversion causes the decision maker to reduce the number of items ordered in order to incur in lower losses.

Multi-period risk-averse newsvendors. Zhang et al. [2009] consider a risk-averse newsvendor problem that aims at minimizing the expected losses, with a constraint limiting the risk of excessive losses within a specified level. Assuming that the demand is a continuous random variable, the authors analyze, first, a single-period problem and derive two optimization models where the above-mentioned risk constraint is formulated using VaR and CVaR, respectively. Subsequently, they extend their analysis to a multi-period setting, and solve the optimization model with the CVaR constraint by means of a sample average approximation method. Computational experiments are carried out using MATLAB to solve the sample average approximated model.

Price-dependent demand. Chen et al. [2009] address a variant of the classical newsvendor problem where the demand, besides being random, is price-dependent. Hence, this problem calls for the simultaneous determination of both the optimal order quantity and the optimal pricing policy. The CVaR, to be maximized, is used to capture the risk-aversion of the decision maker and is a function of the worst profit realizations. For a general demand model, they derive the optimal order quantity for a given price, and, on the other side, they derive the optimal price for a deterministic demand function. Subsequently, the authors analyze additive and multiplicative demand models, providing sufficient conditions for the uniqueness and existence of the optimal pricing and ordering decisions. An interesting result to highlight is that the optimal price of a risk-averse newsvendor is not larger than that of a risk-neutral; and the more risk-averse the decision maker, the smaller the price. The previous research is further extended in Xu [2010] by including the possibility for the newsvendor to place an emergency order once the demand is realized, when the actual demand exceeds the quantity initially ordered. The analysis provided by Xu [2010] follows the same scheme of Chen et al. [2009], and most of the results obtained in the latter article carry over to the case with emergency order possibilities.

Shortage costs. Xu and Li [2010] study the impact of shortage costs on the optimal ordering decisions. To this aim, they consider two risk-averse newsvendor models: the first maximizes the CVaR of the profit realizations; whereas the second maximizes a linear combination of expected profits and CVaR. The authors study these two models both when shortage costs are zero and when they are positive, deriving analytically, in each case, the corresponding optimal order quantity. Their findings reveal that the optimal order quantity is increasing in the shortage cost for both the newsvendor models considered. Wu et al. [2013b] study the impact of uncertain shortage costs on the optimal decisions taken by risk-neutral and risk-averse newsvendors, where risk-aversion is captured using, alternatively, the VaR and the CVaR. The authors derive, under some assumptions on the demand and shortage costs distributions, analytical expressions of the optimal order quantities for each of the considered newsvendor models, and show that the uncertainty in the shortage costs does not affect the optimal order quantity of both the risk-neutral and the newsvendor using the VaR criterion, whereas it results in an increase of the quantity ordered using the CVaR criterion.

Selective risk-averse newsvendors. Chahar and Taaffe [2009] study the problem faced by a company that has to decide whether it should pursue highly profitable, but risky, orders over less profitable, but possibly more stable ones. Simultaneously, the company has also to decide the optimal quantity to procure from its supplier. Hence, in this problem the expected profit is not only influenced by demand uncertainty, but also by the set of demands that the company decides to satisfy. The above problem is studied for a risk-averse newsvendor that aims at maximizing the CVaR, computed as a function of the worst profit realizations. The authors cast the problem as a Mixed-Integer Linear Programming (MILP) model with binary variables, representing the company choices to select or reject the demands. Computational experiments are conducted solving the MILP model with CPLEX on a set of instances randomly generated by the authors. The results reported focus on showing the impact on the solution of introducing the CVaR criterion, and on providing some insights on how certain problem parameters influence the expected profit, the demands selected, and the quantity procured. The authors also devise two other MILP models including the CVaR: the first maximizes a linear combination of the expected profit and the CVaR, whereas the second maximizes the expected profit and imposes limit on the value of the CVaR. Another selective newsvendor problem is investigated by Abdel-

Aal and Selim [2017]. They consider a multi-product problem where the newsvendor has to choose, among a set of potential markets, which markets to serve and what product to sell in each of them. The authors study this problem for three different market entry scenarios, which are related to the type of costs the newsvendor has to pay to enter a market. After proposing a non-linear MIP formulation for each case, the authors reformulate these models as conic quadratic MIPs, and propose problem-specific heuristics for their solution. Some computational results are given where the performance of the proposed heuristic algorithms is compared with that of BARON, for the non-linear MIPs, and that of CPLEX, for the conic quadratic formulations.

Unreliable suppliers. Wu et al. [2013a] study a newsvendor problem where the uncertainty is not only related to the demand, but also to the quantity that the supplier will be able to provide. They derive analytical optimal solutions for three different newsvendor models: the first is a classical risk-neutral newsvendor, whereas the second and the third are risk-averse decision makers that use VaR and CVaR, respectively, to capture the risk preferences. Regarding the latter type of newsvendor, the authors propose two unconstrained optimization models: one that considers only the CVaR as a function of the worst profit realizations, and the other that optimizes a linear combination of the expected profit and the CVaR. Their analytical findings, supported by some numerical examples, highlight that the risk-neutral newsvendor is not affected by the capacity uncertainty. On the other side, the authors show that capacity uncertainty decreases the optimal order quantity using the CVaR criterion.

Competitive risk-averse newsvendors. In the classical settings, the decision maker is assumed to behave as a monopolist. Wu et al. [2014] drop this assumption and consider a competitive environment, seeking to investigate how competition affects the optimal order and pricing decisions. Given an unconstrained model maximizing the CVaR of the worst profit realizations, the authors derive, under certain conditions, analytical optimal order quantities when the total demand is allocated among competing newsvendors using specific splitting rules, and, separately, when newsvendors use prices to compete for the customer demands. Their results indicate that competition tends to lead to overstocking, and that introducing some risk-averse criterion, like the CVaR, can help limiting this effect.

4 Supply chain management

Supply chain management is an area that is attracting an ever growing interest, both from academics and business managers. It refers to the entire process of getting material from suppliers, transforming them into finished products, and ultimately providing the finished products to the customers, possibly through retailer outlets. Modern supply chains are becoming more complex and vulnerable to risks resulting from many potential sources. Often, these risks are classified into two categories (Goh and Meng [2009]): disruption risks, i.e., risks external to the supply chain (such as terrorism, or natural disasters), and operational risks, i.e., risks arising from within the supply chain (such as demand or supply uncertainty). In the remainder of this section, we use this approach to classify the papers that employ the CVaR to incorporate risk-aversion into a decision model for supply chain management. The main characteristics of each paper are summarized in Table II.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Sawik [2011a]	√	√		SPer; MILP
Sawik [2013a]	√			MPer; MILP
Sawik [2013b]	√	√		SPer; MILP
Sawik [2014]	√			MPer; MILP
Cheng et al. [2009]	Lower-level			Bilevel programming; CFS
Caliskan-Demirag et al. [2011]	Retailer			Game theoretical approach
Yang et al. [2009]	Retailer			Game theoretical approach
Ma et al. [2012]	Retailer			Game theoretical approach
Hsieh and Lu [2010]	Retailers			Game theoretical approach
Goh and Meng [2009]		√	√	NLP; Sample average approximation
Xu et al. [2013]	Lowest-level			Tri/bilevel programming
Wu et al. [2010]	√			CFS
Sawik [2011b]	√	√		SPer; MILP

[a] SPer: Single-Period; MILP: Mixed-Integer Linear Programming; MPer: Multi-Period
CFS: Closed-Form Solutions; NLP: Non-Linear Programming

Table II: References in supply chain management problems: A summary of the main characteristics.

Disruption risks. The problem of selecting a portfolio of suppliers under the risk that disruption events may occur has been tackled in a series of papers by Sawik [2011a, 2013a,b, 2014]. At its core, in this problem a set of customer orders is given, and the decision maker needs to decide from which suppliers to procure the parts, along with the respective quantities, which are assembled to satisfy customer orders, with the goal of minimizing the total cost and mitigating the impact of disruption risks. Disruption events may concern, for instance, the quality of supplied materials, the reliability of on-time deliveries, or even that the supplier is not able provide the products, so that the decision maker is asked to choose whether it should cooperate with low cost, but often risky, suppliers over more expensive, but possibly more reliable, suppliers. Two types of disruption scenarios are considered in Sawik [2011a, 2013a,b]: independent local disruptions that may occur at individual suppliers, and global disruptions that may involve all suppliers simultaneously. Sawik [2011a] proposes two MILP models for the above problem employing the CVaR to control the risk of disruptions. One formulation minimizes the CVaR of the total cost, whereas the other minimizes a weighted combination of the expected total cost and the CVaR. Sawik [2013a] extends the previous research to a multi-period setting, where an additional decision is how to schedule the customer orders over the planning horizon. Another extension of the problem addressed in Sawik [2011a] is investigated in Sawik [2013b]. The extension consists of introducing the possibility of selecting some suppliers to be protected against disruptions, and to allocate emergency inventory of products at these suppliers so as to maintain uninterrupted supplies in case of disruption events. Sawik [2014] considers, in a multi-period setting, a further disruption scenario, called semi-global, that is the possibility that a disruption event affects all suppliers in a given geographical region. Besides a MILP model minimizing the CVaR of the total cost, the author proposes another MILP model employing the CVaR that, in this case, is related to the level of service provided to the customers. Some computational results are given in each of the above papers, where the optimization models introduced are solved either with CPLEX or Gurobi. The foremost outcome of these experiments is that, compared to risk-neutral models, the suppliers associated with the highest disruption rates are rarely selected or are selected for smaller fractions of the demand.

Operational risks. Cheng et al. [2009] study the impact of introducing risk-aversion on the optimal decisions of a manufacturer and a retailer belonging to the same two-stage supply chain. The authors propose two bilevel programming models in which the manufacturer is the decision maker of the upper-level, whereas the lower-level decisions are related to the retailer. In their framework, the retailer is the only decision maker that expresses a risk-averse behavior, which is captured inserting the CVaR in the lower-level objective function. By using results derived in Gotoh and Takano [2007], Cheng et al. [2009] transform the proposed bilevel programming formulation into a single-level optimization problem, and obtain analytical solutions under the assumption that the demand is uniformly distributed. Some numerical examples are reported to illustrate the proposed models, especially in terms of the interactions between the manufacturer and the retailer and the impact of considering the risk-aversion of the retailer. Caliskan-Demirag et al. [2011] complement the previous research by analyzing rebate promotions. More specifically, a risk-neutral manufacturer can offer customer and/or retailer rebates, whereas the retailer is a risk-averse decision maker employing the CVaR. Assuming that the demand is random and price-dependent, the authors analyze the manufacturer's rebate amount decisions and the retailer's joint inventory and pricing decisions applying a game theoretical approach. Caliskan-Demirag et al. [2011] derive some structural properties relating the optimal decisions of the manufacturer and retailer and, in two special cases, characterize the existence of a Nash equilibrium. Yang et al. [2009] study the problem of coordinating a supply chain consisting of a risk-neutral supplier and a risk-averse retailer. In general terms, given a supply contract provided by the supplier, the retailer chooses an order quantity that maximizes the CVaR of her worst profit realizations. The authors model this problem as a Stackelberg game in which the supplier is the leader and the retailer is the follower. Yang et al. [2009] examine four types of supply contracts and, for each of these types, identify a set of properly designed contracts which can lead to a coordinated supply chain. Ma et al. [2012] employ a Nash-bargaining approach to model the bargaining process for wholesale price and order quantity in a two-stage supply chain composed of a risk-neutral manufacturer and a risk-averse retailer using the CVaR. Assuming that the random demand is distributed uniformly, the authors show that there exists a Nash-bargaining equilibrium both when the two logistics operators have equal and unequal bargaining powers. Subsequently, Ma et al. [2012] extend their analysis to consider a price-dependent random demand. Some numerical results are reported where the random demand is assumed to be normally distributed. A game theoretical approach is also applied by Hsieh and Lu [2010] to study a two-stage supply chain in which one risk-neutral manufacturer sells products through two risk-averse retailers, under a price-dependent random demand. Their analysis focuses on determining the manufacturer's return policy when the retailers are engaged or not in horizontal price competition. The manufacturer acts as a Stackelberg leader which has to determine the unit return price. On the other side, each retailer has to decide the selling price to the customers and the quantity ordered to the manufacturer when there is horizontal price competition, and simply the quantity ordered when no competition occur. The authors report some numerical examples that show how, under different settings, retailers' risk-aversion impacts the manufacturer's and retailers' decisions.

Goh and Meng [2009] discuss a stochastic programming formulation for a three-stage supply chain consisting of two suppliers, one manufacturer and two distributors. The main uncertain factor is the distributor demands, which is captured into a non-linear programming model by means of the CVaR. The proposed optimization model maximizes a convex combination of the expected profit of the manufacturer and the CVaR of the losses, subject to a constraint, among others, setting an upper limit on the CVaR. After devising a sample average approximation method for solving the above optimization model, the authors report some computational experiments, carried out using MATLAB, applied to a wine industry supply chain problem. Xu et al. [2013] also study a three-stage supply chain, here comprising one supplier, one manufacturer, and one retailer. Based on the assumptions that the demand is random and the retailer is the only risk-averse decision maker, the authors design a trilevel programming model where the objective functions of both supplier and manufacturer maximize their own profits, whereas the objective of the retailer maximizes the CVaR of her worst profits. This trilevel programming model is reduced to a more tractable bilevel programming one using results taken from the literature on the newsvendor problem, in particular those reported in Gotoh and Takano [2007].

Wu et al. [2010] consider a flexible supply contract with a commitment-options structure, and study the impact of risk-aversion on the optimal ordering decisions. In general, these contracts are agreements between a supplier and a manufacturer, such that the latter can make procurement decisions at two time moments. Before the random demand is realized, the manufacturer decides the quantity to order and reserve the option to procure a given quantity of products in a future moment. When the demand materializes, the manufacturer makes the final decision whether or not to exercise the option and how much of the option to order. Wu et al. [2010] propose an unconstrained optimization model that maximizes the CVaR of the manufacturer's profits, and derive closed-form solutions for the optimal ordering strategy. Some computational experiments are reported to especially show the impact of risk-aversion on the manufacturer's optimal decisions.

Finally, Sawik [2011b] tackles a problem similar to the one addressed in Sawik [2011a], mentioned above. The main differences are that in Sawik [2011b] no disruption scenarios are considered, but the CVaR is applied to control the impact of delays when on-time deliveries may be unreliable.

5 Transportation and traffic control

This section provides an overview of the research where the CVaR has been employed in transportation and related problems. More specifically, in the following we review papers that deal with the problem of transporting hazardous materials from their origins to their destinations. Subsequently, we analyze papers that address some, broadly speaking, related problems, such as fleet management, maintenance of transportation infrastructures, and traffic control problems. The main characteristics of each paper are summarized in Table III.

Transporting hazardous materials. While transporting hazardous materials (hazmat), a critical issue for a decision maker is the determination of the best possible route to transport the shipment from its origin to the destination. Models in this context usually focus on measuring the risk resulted by following a specified

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Toumazis et al. [2013]	√			
Toumazis and Kwon [2016]	√			Worst-case CVaR
Toumazis and Kwon [2013]	√			Time-dependent network
Faghih-Roohi et al. [2016]	√			Time-dependent network; Multi-commodity
Ansaripoor et al. [2014]		√		MILP
Ansaripoor et al. [2016]		√		Multi-stage stochastic MILP
Seyedshohadaie et al. [2010]	√			LP
Yin [2008]	√			Non-linear programming
Zhang and Yin [2008]	√			MILP

[a] LP: Linear Programming; MILP: Mixed-Integer LP

Table III: References in transportation and traffic control problems: A summary of the main characteristics.

path in a network, where risk is related to the expected consequences that an accident can cause on both the population and the environment. Toumazis et al. [2013] address the problem of determining how to transport hazmat from one origin to one destination over a directed and weighted graph, where each arc has two attributes: the probability that an accident occurs, and an estimate of the consequences. In their approach, an optimal solution is a path, i.e., a sequence of arcs having the above attributes, with minimum CVaR. Moreover, the CVaR of a path is related to the accident consequences that may occur in all the arcs traveled while following that path. Some numerical results are given for a case study applied to the city of Albany (NY, USA). Motivated by the observation that historical data are insufficient, and, as a consequence, the accident probabilities and accident consequences have to be treated as uncertain data, Toumazis and Kwon [2016] extend the research presented in Toumazis et al. [2013] by considering the worst-case CVaR. The authors assume that each of the two uncertain parameters belongs to a box-constrained uncertainty set, and devise a solution method that works along the lines of the one presented in Toumazis et al. [2013]. The results of a case study applied to the city of Buffalo (NY, USA) are presented. Toumazis and Kwon [2013] study a time-dependent extension of the problem introduced in Toumazis et al. [2013]. Particularly, they consider a time-dependent network, such that the probability of an accident and the resulting consequences depend on the shipment's entrance time in the arc, mainly due to traffic conditions. Therefore, compared to the two previous papers, in Toumazis and Kwon [2013] an additional decision about the optimal departure time of the shipment has to be taken. The authors report a case study applied to the city of Buffalo (NY, USA). Faghih-Roohi et al. [2016] extend the research reported in Toumazis and Kwon [2013] to a multi-commodity setting by assuming the presence of different types of hazmat. This assumption implies that the probability of an accident and the resulting consequences associated with each arc depend also on the type of hazmat transported. Furthermore, the decision maker has also to determine the priority among hazmats for transportation. Some numerical examples are reported, both assuming that different hazmats cannot and can be transported simultaneously across the network. In each of the previous papers a solution procedure is devised which works along the same general lines. Indeed, all procedures are based on solving a finite number of shortest path problems, and have been implemented in MATLAB.

Fleet management. Ansaripoor et al. [2014] analyze the problem faced by a firm that aims at replacing some of its vehicles with fossil fuel (petrol and diesel) and electric vehicles. Particularly, the authors consider a fleet manager that aims at achieving a balance between expected costs of the new vehicles over a planning horizon and risks incurred, taking into account the uncertainties related to carbon and fuel prices, mileage driven, and fuel consumption. This problem is cast as a MILP that minimizes a linear combination of the expected costs and the CVaR, the latter computed as a function of the costs incurred for the vehicles chosen. After providing some mathematical conditions under which the CVaR of electric vehicles is smaller than that for fossil fuel vehicles, the authors present a case study based on real data. As a general conclusion of their study, the authors report that choosing electric vehicles can significantly mitigate risk exposure compare to fossil fuel ones, but at additional expected costs. The above research has been extended to a multi-period setting by the same author in Ansaripoor et al. [2016]. Furthermore, compared to Ansaripoor et al. [2014] the authors consider additional vehicle technologies (e.g., hybrid vehicles), assume that the demand for vehicles in each year is stochastic, and propose a time-consistent version of the CVaR (called Recursive Expected CVaR) to manage the multi-period setting. The resulting problem is formulated as a two-stage stochastic MILP model, which is solved using CPLEX.

Maintenance problems. Seyedshohadaie et al. [2010] tackle the problem of determining optimal maintenance and rehabilitation decisions for transportation infrastructure that take into consideration the risk associated with the uncertain deterioration of these facilities. In Seyedshohadaie et al. [2010], the latter is measured using the CVaR. For short-term decisions concerning how to allocate resources under budget constraints, the authors present two LP optimization models: one minimizes the largest CVaR among all infrastructures, whereas the other minimizes the sum of CVaRs over all infrastructures. As an example of application of their methodology, they investigate the problem of pavement roughness.

Traffic control. Traffic signal timing refers to the determination of who has the right-of-way at an intersection. The environment where these decisions are taken is intrinsically not deterministic, since traffic flows are, by their nature, uncertain. Consequently, an issue that decision makers may be confronted with is to determine what flows to use while optimizing signal timings. In this context, Yin [2008] investigates the problem of determining a signal timing plan whose performance is near-optimal in an average sense, and sufficiently stable under any realization of the uncertain traffic flows. To model this problem, the author presents three alternative mathematical formulations, one of which minimizes the CVaR computed as a function of the delay incurred per vehicle. The resulting formulation is non-convex but can be solved using some routines available in optimization packages, such as MATLAB. The author compares the timing plans resulting from optimizing the three proposed optimization models plus other more conventional solutions, and assesses the effectiveness of each solution using a Monte-Carlo simulation to mimic real-world traffic conditions. The results show that the model using the CVaR is able to generate timing plans that are less sensitive to fluctuations in traffic flows. Zhang and Yin [2008] study the problem of determining the appropriate signal settings when actuated signals (such as, push buttons and loop detectors) are employed. As a matter of fact, in this application the start and end of greens are not deterministic, making red times uncertain. Zhang and Yin [2008] extend a classical optimization model proposed by Little by incorporating the CVaR to capture this type of uncertainty. The proposed model is a MILP that minimizes the CVaR computed as a function of the bandwidth that, in general terms, can be described as the amount of time available for vehicles to travel through a corridor. Some computational experiments are conducted implementing the above optimization model in GAMS and solving it with CPLEX. Finally, the performance of the solutions obtained is evaluated simulating several samples of red times.

6 Location and supply chain network design

Location science is a very active research area, as proved by the ever growing body of related literature. This academic interest is motivated by the observation that facility location problems naturally arise in several different application settings, including supply chain management, distribution, transportation, and telecommunication, to name a few, and play a crucial role in strategic planning for both the public and private sectors. The location of facilities is a strategic and long-term decision that has to be made under considerable uncertainty with respect to market development and changing economic environments. Indeed, some decision parameters (e.g., customer demands, supplying costs) may change dramatically once the decisions have been employed. Furthermore, these decisions are, once implemented, usually difficult or impossible to reverse. Location decisions are typically faced also in supply chain network design problems, i.e., problems that aim at determining the physical configuration and infrastructure of the supply chain. Key decisions in this context involve the number, locations, and capacity of manufacturing plants and warehouses, the assignment of retail outlets to warehouses, as well as strategic investment decisions aimed at modifying the current infrastructure of the supply chain.

In this section, we review the foremost papers tackling a problem belonging to the classes mentioned above, and that incorporate the CVaR into their optimization framework. A summary of the main characteristics of each paper reviewed in this section is reported in Table IV.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Chen et al. [2006]	√			*p*-median; MILP
Noyan [2012]		√		Two-stage stochastic MILP
Givler Chapman and Mitchell [Forthcoming]		√		Deterministic; Non-linear MIP
Soleimani and Govindan [2014]		√		Two-stage stochastic MILP
Soleimani et al. [2014]		√		Two-stage stochastic MILP
Claro and de Sousa [2010]	√			BOMILP; VNS; VNS+TS
Claro and de Sousa [2012]	√			BOMILP; VNS+TS
Tomlin and Wang [2005]	√			LP
Carneiro et al. [2010]			√	Two-stage stochastic LP

[a] LP: Linear Programming; MI(L)P: Mixed-Integer (L)P; BOMILP: Bi-Objective MILP
VNS: Variable Neighborhood Search; TS: Tabu Search

Table IV: References in location and supply chain network design problems: A summary of the main characteristics.

***p*-median problem.** Chen et al. [2006] study a *p*-median problem where distances between facilities and customers as well as customer demands are uncertain. To address this problem, the authors assume that the uncertain parameters can be modeled by a discrete set of scenarios and, consequently, propose a scenario-based MILP model that aims at minimizing the CVaR of the regret, a metric often used in decision-making under uncertainty. Note that in this paper the CVaR is usually called the α-reliable mean-excess regret. Computational results are given where the uncertainty is related only to customer demands, and solving the corresponding optimization model with CPLEX.

Disaster management. Noyan [2012] studies the problem of determining the location of the response facilities and, for each of them, the inventory levels of the relief supplies in the presence of uncertainty related to the occurrence of a disastrous event. Indeed, when a disaster occurs, roads and facilities may be damaged and, therefore, the transportation capacities and the available amount of supplies may vary according to the severity of the disaster. Furthermore, the demand of various commodities arising at each location is also affected by such type of events. After detailing a modeling and solution framework from a general perspective, the latter is applied to the context of disaster management. The resulting formulation is a two-stage stochastic MILP model where the first-stage decisions are types and locations of facilities, along with the amount of each commodity to be stocked at each facility. The second-stage decision variables are concerned with distributing the supplies to satisfy the realized demand. In this formulation, the CVaR appears as a weighted term into the objective function, and is a function of some uncertain costs, including salvage costs (if there is a surplus), penalty costs (if there is a shortage), and shipping costs. Some computational results are presented on a case study, taken from the literature, concerning hurricane threats in southeastern US. The author concentrates on showing that the location and allocation decisions change modifying the risk preferences of the decision maker by changing the value of α in the CVaR and the weight of the risk term in the objective function. Some further computational results are given to assess the effectiveness of the proposed decomposition algorithms. A relevant example of applying the CVaR in a deterministic environment is reported by Givler Chapman and Mitchell [Forthcoming]. The application context of this research is humanitarian logistics, and in particular the organization of the relief operations following a major disaster. The problem calls for the optimal location of relief centers, which are facilities where supplies are hold, and allocation of people living in different centers to each of the open facility. The CVaR here is used to determine a distribution of relief centers that minimizes the inconvenience for the percentage of population that would otherwise incur the largest walking distances. The authors cast this problem as a non-linear MIP, whose objective function aims at minimizing a weighted combination of the total operational costs, the CVaR of walking distances, and the average walking distance for the whole population. Computational experiments are conducted solving the above optimization model combining a golden section search method with CPLEX.

Supply chain network design. Soleimani and Govindan [2014] address the problem of optimally designing and planning a reverse supply chain network when the demand of return products (i.e., the used products) as well as their prices are uncertain. In the design stage the location of facilities is decided, whereas at the planning level the flows between all entities involved in the network are determined. The authors cast this problem as a two-stage stochastic MILP model, incorporating into the objective function the CVaR of various costs related to the functioning of the reverse supply chain. Some computational results are given where the proposed optimization model is solved with CPLEX. In a related research, Soleimani et al. [2014] address the problem of designing and planning a closed-loop supply chain network, where forward and reverse flows of products are considered simultaneously. The authors assume that the demand and prices of both forward and return products are random. Soleimani et al. [2014] develop a two-stage stochastic MILP model, and embed into it three alternative risk measures, namely the VaR, the CVaR, and the mean-absolute deviation. The impact of using the latter three risk measures is analyzed conducting some computational experiments, which show the excellent properties of the CVaR compared to the other two measures.

Strategic investment planning. By their nature, investment planning involves long-term, often irreversible, strategic decisions. Claro and de Sousa [2010] study a multi-stage capacity expansion planning problem, when uncertainty affects the demand and the investment costs for achieving the expansion. They consider a discretized planning horizon over which the evolution of demand and investment costs is represented through a scenario tree. The problem is then cast as a bi-objective MILP model: one objective minimizes the expected value of the total investment cost, whereas the second objective is the minimization of the CVaR of the total investment cost. To solve this problem, the authors devise multi-objective versions of a Variable Neighborhood Search (VNS) and a hybrid heuristic combining a Tabu Search (TS) and a VNS. Computational experiments are given for a set of randomly generated instances, where the solutions obtained with the two above heuristics are compared with the efficient solutions computed using CPLEX embedded into a classical ε-constraint method. The results indicate that the hybrid heuristic combining TS and VNS produces, on average, the best results. In a later article, Claro and de Sousa [2012] extend the previous research by including multiple products and flexible resources, i.e., the capacity available of some resources can be dedicated to produce several products. This extension introduces an additional level of decisions related to how much capacity of each resource has to be used to produce each product. Based on the results reported in Claro and de Sousa [2010], Claro and de Sousa [2012] adapt the mathematical formulation and the hybrid heuristic to the problem at hand. Tomlin and Wang [2005] study the performance of an unreliable supply chain where several products are negotiated. They consider a firm that can invest in product-dedicated resources, or in flexible resources that can produce all products, or in a combination of both. These decisions are affected by the uncertainty related to product demands and the unreliability on the supply side (i.e., the realized resource investment may fail, and hence differ from its expected value). Tomlin and Wang [2005] consider four different supply chain design layouts, and different firm's risk attitudes that lead to different optimization problems. One of these problems aims at maximizing the CVaR of the firm's terminal wealth, which is the random gain or loss achieved by the firm depending on the resource investment decisions. For the case of discrete random variables, they introduce LP models and conduct some computational experiments in order to compare the different supply chain design layouts and their sensitivity to some problem elements, including the firm's risk attitude and the reliability of the resources. Carneiro et al. [2010] consider an entire oil supply chain and, in this application context, study the problem of determining a portfolio of investments, in both refineries and logistical infrastructures, in the presence of uncertainty. The authors consider as sources of uncertainty the crude oil supply, the demand for final product, and oil and product prices. The above problem is cast as a two-stage stochastic LP model that maximizes the net present value of the supply chain, imposing a lower bound on the value of the CVaR, and including several application-related constraints. The authors present a case study applied to an oil supply chain located in Southeast Brazil, solving the proposed optimization model with CPLEX.

7 Networks

Many important optimization problems can be analyzed by means of a network representation. As a consequence, network models are possibly one of the most important special structures considered in operations research. In this section, we review the main contributions in the literature dealing with network problems which incorporate the CVaR, which is used in the majority of the cases to capture possible arc or edge failures. Table V provides a summary of the main characteristics of each paper reviewed in this section.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Commander et al. [2007]			√	Deterministic; MILP
Boginski et al. [2009]			√	AEF; LP
Sorokin et al. [2013]			√	AEF; MILP; Heuristic algorithm
Ma et al. [2016]			√	AEF; MILP; B&C
Mahdavi Pajouh et al. [2017]			√	AEF; MILP; Decomposition algorithms
Yezerska et al. [Forthcoming]			√	AEF; LP; TS; GRASP
Kalinchenko et al. [2011]	√			MILP
Rysz et al. [2014]	√			MILP; B&B

[a] LP: Linear Programming; MILP: Mixed-Integer LP; AEF: Arc/Edge Failure; B&C: Branch-and-Cut; TS: Tabu Search GRASP: Greedy Randomized Adaptive Search Procedure; B&B: Branch-and-Bound

Table V: References in network problems: A summary of the main characteristics.

Deterministic applications. Given a network composed of communication nodes to be jammed and potential locations for jamming devices, Commander et al. [2007] study the problem of determining the optimal number and positions for these devices, in order to neutralize the wireless communication network of an enemy. It is worth noting that here the application context is completely deterministic. The authors propose two formulations requiring that each communication node must be jammed. Based on the observation that it may often be sufficient to jam some percentage of the total number of communication nodes in order to acquire an effective control over the network, they extend the previous two mathematical formulations adding percentile risk constraints. These constraints are modeled by interpreting each communication node as a random scenario, and then applying the CVaR in the traditional way. The resulting formulation is a MILP model. Computational results are given for two case studies, and focus on comparing the different solutions obtained solving with CPLEX the optimization models without and with percentile constraints.

Network flow problems. Boginski et al. [2009] extend the classical minimum-cost flow problem to consider the possibility that some arcs of a network can randomly become unusable. As motivating applications, the authors report the problem of maximizing traffic flows along highways which are subject to extreme congestion, and hence causing arc failures, and the routing of vehicles in a battlefield scenario since, in this context, the probability of a road being unusable due to an ongoing fire fight renders flow along that arc impossible. In their approach, the CVaR measures the worst possible losses of flow in the network, and is included in an LP model as a constraint limited by an upper bound. The authors present an illustrative numerical example that shows the differences between the solution obtained using the classical (deterministic) minimum-cost flow problem and that obtained using the optimization model with CVaR constraint (solved with the XPRESS solver). A problem closely related to the minimum-cost flow is the fixed-charge network flow problem, which is another classical topic studied by operations researchers. Sorokin et al. [2013] investigate the latter problem when there might be uncertain arc failures. Similar to Boginski et al. [2009], also in the current paper the CVaR is employed as a constraint to limit the risk that large losses of flow in the network occur. The resulting formulation is a MILP model. To solve the problem, Sorokin et al. [2013] extend a heuristic algorithm previously developed for the deterministic case. Computational experiments are given solving randomly generated instances, and focus on validating the performance of the heuristic algorithm against CPLEX.

Minimum-cost spanning problems. The minimum spanning k-core problem calls for determining a minimum-cost spanning subgraph such that the degree of every node is at least k. Ma et al. [2016] study this problem when some edges can become unusable. It has applications in network design problems where a set of hubs has to be connected in such a way that the resulting network has some desirable properties. Examples of these networks are transportation networks where hubs are represented by transshipment points, logistics networks with distribution centers, or airline networks with major airports. In these applications, it is important that the network designed is able to withstand limited interhub link failures, so that the connection is not interrupted even if a link fails. Ma et al. [2016] cast the problem as a MILP where the CVaR is employed to capture violations of the above-mentioned minimum k degree requirement. After describing a reformulation of the latter optimization model, the authors devise a Branch-and-Cut (B&C) algorithm for its solution. Computational experiments are conducted on randomly generated instances, and the results reported focus on validating the performance of the proposed B&C algorithm against the Gurobi solver.

Special network structures. Graph models of data are techniques used in graph-based data mining with applications in biological and social networks. In reality, these graph models are subject to errors in capturing the relationships (edges) between different data entities (nodes). The resulting network is, therefore, subject to the possibility that some edges are not present. In this context, Mahdavi Pajouh et al. [2017] study the problem of detecting large 2-clubs subject to edge failures, where a 2-club is a special graph structure used to denote a cluster of entities with strong relationships. The CVaR is employed to limit the risk of losing, due to edge failures, the 2-club property in the detected clusters. The resulting formulation is a MILP model, where the CVaR is constrained by an upper bound. The authors devise two decomposition algorithms for the solution of the problem, and report on some computational experiments comparing the performance of the two algorithms solving randomly generated test problems, as well as real-life biological and social network instances. Along a similar line of research, Yezerska et al. [Forthcoming] study the maximum clique problem when multiple edges may randomly fail. Cliques are other special graph structures used in cluster detection techniques to identify cohesive subgroups in a social network. Yezerska et al. [Forthcoming] highlight that sometimes the edges of a network are not known with certainty, for instance, when networks are constructed using data extraction processes that are potentially error-prone. The authors use the CVaR to limit the risk of losing the clique property as a result of edge failures. To solve the problem, they adapt three well-known heuristic algorithms: a Tabu Search (TS); a variant of TS proposed for finding stable sets; and a Greedy Randomized Adaptive Search Procedure (GRASP). Additionally, they extend an exact approach available in the literature for the standard maximum clique problem by including a procedure that, whenever necessary, verify that the clique found satisfy the CVaR requirement. Computational results are reported for a set of test problems generated by the authors from some benchmark graph instances. Kalinchenko et al. [2011] study the problem of optimally scheduling dynamic sensors in the presence of uncertainty. These sensors are used for area surveillance and move over a network of nodes to be monitored at every discrete time moment, and where the number of nodes is considerably larger than the number of sensors available. The main decisions in this problem concern which nodes to monitor at every time moment. The uncertainty is related to the information losses that occur when nodes are not under surveillance at certain time moments. To model this uncertainty using the CVaR, the authors employ some random penalties that measure the information loss occurred at each node in each time period. The problem is cast as a MILP model that minimizes the CVaR, which measures the average of the largest penalties. Subsequently, to ensure the connectivity between nodes of the network, the authors extend the above formulation by including constraints that define some special network structures, such as 2-club and k-plex. Computational experiments are conducted comparing the performance of CPLEX with that of a software package called AORDA Portfolio Safegard while solving the proposed optimization models. The results show that CPLEX outperforms, on average, the latter optimization software.

Uncertain node weights. In contrast with the papers mentioned previously, the uncertainty considered in the article by Rysz et al. [2014] is associated with the weight of the nodes of a network rather than the possible failure of edges. More specifically, Rysz et al. [2014] discuss a class of maximum weighted subgraph problems with uncertain weights. In general terms, problems in this class call for the determination of a subgraph with the largest sum of stochastic weights and that preserve a given hereditary property. The authors propose a general mathematical formulation for this class of problems, using a family of higher-moment coherent risk measures which encompass the CVaR as a special case. When the CVaR is minimized, the resulting formulation is a MILP model. Some computational experiments are reported choosing the maximum weighted clique problem as a case study. The proposed model is solved with CPLEX and a Branch-and-Bound (B&B) algorithm specifically designed to exploit some properties of the problem structure.

8 Scheduling

In scheduling problems, uncertainty may be associated with job processing times. Although the use of CVaR to capture uncertainty in scheduling problems seems a promising topic, only two relevant papers appeared in the literature, each one related to a specific problem. The main characteristics of each paper are summarized in Table VI.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Sarin et al. [2014]	√			MILP; Benders Decomposition
Chang et al. [2017]	√			AP + SOCP; Heuristics

[a] MILP: Mixed-Integer Linear Programming; AP: Assignment Problem; SOCP: Second-Order Cone Programming

Table VI: References in scheduling problems: A summary of the main characteristics.

Total weighted tardiness problem. Sarin et al. [2014] argue that the CVaR has a tendency to simultaneously optimize both the expectation and the variability of the chosen performance measure. Additionally, if the expectation can be represented by a linear expression, then CVaR retains such linearity. A scenario-based MILP formulation that minimizes the CVaR in a general scheduling problem is proposed, and an L-shaped (Benders) decomposition procedure is designed for its exact solution. The approach is applied to the single machine total weighted tardiness problem, where processing times are uncertain. For large scale instances, a problem-specific dynamic programming-based heuristic is also designed. The effectiveness of the CVaR criterion is also shown for the parallel machine total weighted tardiness problem.

Total completion time problem. Chang et al. [2017] consider a single machine scheduling problem where job processing times are uncertain, but their averages and standard deviations are known. The objective is to minimize the CVaR in terms of the sum of completions times or total flow time. Due to the special structure of the problem considered, an equivalent formulation is obtained where the minimum of two objective functions is taken: the first objective is proportional to the expected total flow time, whereas the second objective is a combination of the expected total flow time and its standard deviation. Minimizing the first objective leads to a deterministic problem, solvable by sorting techniques; minimizing the second objective leads to a complex second-order cone programming problem. For the special case of uncorrelated processing times, three heuristic polynomial-time algorithms based on a Cauchy relaxation of the objective function are proposed. Computational tests show that: the proposed algorithms find very good solutions in a short computing time; the minimization of the CVaR reduces the variability in performance at the expense of a slight increase in the expected performance; the proposed approach is robust with respect to different distributions of the processing times.

9 Energy

In the energy sector, management problems usually involve two main sources of uncertainty: production rate of renewable resources (e.g., water, wind, solar) and energy price. Several authors propose scenario-based optimization models using CVaR to control the risk associated with uncertain production rates and/or prices. The specific applications include the management of an existing production network, its expansion, and also pricing and procurement issues. Table VII summarizes the main characteristics of the papers reviewed in this section.

Hydro-chain scheduling. Doege et al. [2006] model the operational flexibility of a hydro pump storage plant by using an LP where expected profit is maximized, whereas risk is controlled by a constraint on the CVaR value. The authors show that using this model the risk on water volume, which cannot be hedged with standard contracts from power exchanges, can be managed by a clever dispatch policy. In particular, the optimal values of the dual variables are used to derive the marginal value of risk and the marginal value of volume. García-González et al. [2007] present a model for hydro-chain scheduling in the short-term electric-

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Doege et al. [2006]			√	LP
García-González et al. [2007]			√	MILP; Maximin + CVaR
Catalão et al. [2012]		√		MINLP
Hatami et al. [2009]		√		MINLP; SDP
Pousinho et al. [2011]		√		LP
Yau et al. [2011]	√			Two-Stage Stochastic MILP
Hosseini-Firouz [2013]		√		LP
Moghaddam et al. [2013]		√		MILP
Tajeddini et al. [2014]		√		MILP
Hemmati et al. [2016]		√		MILP
Zheng and Pardalos [2010]			√	Two-Stage Stochastic MILP; BD
Bruno and Sagastizábal [2011]			√	Two-Stage Stochastic LP; BD; LD
Mena et al. [2016]	√			BONLP; Evolutionary Algorithm
Lu et al. [2016]			√	LP
Salahi and Jafari [2016]			√	MILP

[a] LP: Linear Programming; MILP: Mixed-Integer LP; MINLP: Mixed-Integer Non-LP; SDP: Stochastic Dynamic Progr. MPC: Model Predictive Control; BD/LD: Benders/Lagrangian Decomposition; BONLP: Bi-Objective Non-LP

Table VII: References in energy problems: A summary of the main characteristics.

ity market. The company is assumed to be price-taker, and, hence, market prices are considered exogenous and modeled by means of scenarios. The objective function maximizes the expected profit subject to two constraints that model risk-aversion: a Maximin constraint, that imposes a lower bound on the minimum profit over all scenarios, and a CVaR constraint, that limits the average of the lowest profits. Since the produced energy at a hydropower unit is a non-linear function of the turbine discharge (i.e., water consumption) and the net-head of the associated reservoir (i.e., the water pressure at the turbines), the problem is simplified considering the produced energy as a function of water consumption alone. In this way, a MILP formulation can be derived. An iterative procedure is designed to converge to the correct energy/discharge/net-head equation while maximizing the expected profit. At each iteration, the above MILP model is solved. The approach is applied to a realistic hydro-chain, showing the effects of the risk-averse constraints on the distribution of profits. Catalão et al. [2012] propose a non-linear MIP approach to enable optimal hydro scheduling for the short-term horizon, including the effects of head on power production, start-up costs related to the generation units, multiple regions of operation, and constraints on water discharge variation. Given the day-ahead time horizon, uncertainty is restricted to energy market prices, and it is modeled using discrete scenarios. The general model description includes a MILP feasible set and a non-linear objective function, where non-linearities originate from power generation and water head influence. Subsequently, a quadratic (non-convex) function is used. The objective function maximizes a weighted combination of the expected profits and the CVaR of the lowest profits. A realistic case study is considered and solved using MATLAB/CPLEX.

Pricing and procurement. Hatami et al. [2009] consider a pricing and procurement problem for a retailer in the electric power industry. A non-linear MIP model is formulated to determine the optimal sale price and procurement policy, including spot markets, forward contracts, call options and self-production. In particular, the maximized objective function is a weighted difference of the expected profit and the CVaR, computed under an appropriate number of scenarios. In a case study that covers a one-month period, the authors analyze how the weight in the objective function, called risk-aversion parameter, affects the procurement and price policy selected. The model is solved by a two-phase approach, using stochastic dynamic programming techniques. Pousinho et al. [2011] suggest a stochastic programming approach for trading wind energy in a market environment. In their problem, uncertainty affects energy market prices and intermittent nature of wind energy. A deterministic equivalent scenario-based LP model is proposed, where the decision variables are the hourly day-ahead offers by the wind power producer. The objective is to maximize a combination of the expected profits and the profit CVaR. Results for a case study involving 1000 scenarios are analyzed. Yau et al. [2011] propose a mixed-integer stochastic model for a power providing agent that has to select custom electricity contracts and procurement strategies. The model includes binary, non-negative integer, and continuous variables. A two-stage stochastic approach is used. The model minimizes the CVaR of the worst realizations of the difference between costs and revenues. On a set of realistic instances, the results of the model are compared with the results of a risk-neutral version, where the average difference between costs and revenues is minimized. The models are implemented in AMPL and solved by CPLEX 9.0.

Wind farm management. Hosseini-Firouz [2013] addresses the problem of deriving the best offering strategy for a wind power producer in a short-term electricity market. The producer operates in a market where hourly offers have to be made for the next day, based on wind speed forecasts and day-ahead market prices. Immediately before the beginning of the next day, an adjustment market allows to adjust the previous offers given the more accurate wind forecasts. Additionally, during the next day, a balancing market allows to tune every hour the offers. Uncertainty affects wind speed volatility and prices on the three considered markets. A scenario-based LP model is proposed, whose objective is the maximization of a weighted combination of the expected profit and the CVaR of the profits. In a case study, the role of this risk factor is analyzed in detail. Moghaddam et al. [2013] study the problem of deriving the best offering strategy for a generation company that owns a wind farm and a hydro system. This company acts in the same market considered in Hosseini-Firouz [2013] but, with respect to the previous situation, the company may use the hydro system to compensate the error committed while forecasting the wind power. Uncertainty involves the wind speed volatility and the day-ahead prices. A MILP model is formulated, whose objective is the maximization of a weighted combination of the expected profit and the CVaR of the profits. In a related research, Tajeddini et al. [2014] consider a more complex virtual power plant than the one studied in Moghaddam et al. [2013], including a wind farm, a photovoltaic plant, a micro turbine, a diesel generator, and a battery bank. As in Moghaddam et al. [2013] a MILP model is developed, whose objective function is again a weighted combination of expected profit and CVaR. A comparison between the results obtained using CVaR and the results obtained using VaR is also shown, though the presented model includes only CVaR.

Energy storage. Hemmati et al. [2016] study an Energy Storage System (ESS) integrated into a grid including a thermal power generator and a wind power generator. Given the demand pattern related to a certain time horizon, a decision maker has to decide about the profile of charge/discharge powers at the ESS, in order to minimize costs while facing uncertainty about the available wind power. A scenario-based MILP model is proposed where a weighted combination of the expected cost and the CVaR of the same cost is minimized. A number of constraints express power balance, dynamics of the thermal unit, and dynamics of the ESS.

Power network expansion. Zheng and Pardalos [2010] consider an expansion planning problem for a natural gas transmission network. Given a forecast about future demands, the capacities of the arcs in the network have to be chosen among a discrete set of possible sizes. Additionally, the location and size of liquefied natural gas terminals have to be decided. Two stochastic two-stage MILP are designed, with binary variables in the recourse sub-problems. Both models minimize the expected cost, but the second includes risk considerations by imposing a CVaR constraint on the possible shortage of natural gas at the nodes of the network. An embedded Benders decomposition algorithm is proposed, where Benders cuts are applied both in the first and second stage. The algorithm is tested on instances with up to ten thousand scenarios, involving tens of thousands of binary variables. A similar problem is studied by Bruno and Sagastizábal [2011], but their analysis is more generic and does not consider specific application-oriented issues. They consider a two-stage stochastic LP where expected cost is minimized and a CVaR constraint is incorporated to control the volatility of the objective function with respect to a set of scenarios. Two decomposition methods are proposed for large-scale instances: Benders decomposition and Lagrangian decomposition. Both methods are applied on a case study involving a Brazilian energy company. The computational tests suggest that the Lagrangian approach has more practical appeal. The problem of integrating renewable distributed generation into an existing electric power network is considered by Mena et al. [2016]. The main decisions concern the number and position in a network of photovoltaic and wind power plants in order to integrate the production of existing conventional power plants. Uncertainty involves several factors:

solar irradiation, wind speed, energy price, and plant failures. Every source of uncertainty is modeled by appropriate probability distributions (e.g., Gaussian, Beta, Weibull), and scenarios are generated accordingly. The authors consider two objective functions to minimize: the expected total cost and the *CVaR deviation* (DCVaR). The latter measure has been introduced in Rockafellar et al. [2006] as the CVaR of the residuals with respect to the mean. In other words, the DCVaR can be interpreted as a standardized CVaR with respect to the mean. The resulting bi-objective model is non-linear and non-convex. An evolutionary algorithm of the class NSGA-II is developed and applied to a case study. As one may expect, the solution corresponding to the minimum CVaR value, which is the solution with the minimum sum of the two objective considered, lies in the middle of the Pareto front and possesses the most desirable characteristics. However, the proposed bi-objective approach allows to better analyze the trade-off between efficiency and risk. Lu et al. [2016] address a dynamic generation expansion planning problem where it has to be decided the composition of the different energy sources to meet energy demand over time, while taking into account costs and carbon emissions. Three sources of uncertainties are considered: fossil fuel prices; costs of emission reduction technologies; costs of carbon emissions. The authors propose a time-indexed LP model that aims at maximizing the expected net profit. A CVaR constraint is introduced to control the risk of possible losses, considered as the opposite of the net profits. Scenarios on the net profit are obtained from Monte Carlo simulation applied to the three sources of uncertainties. A case study involving a time horizon of 20 years is built for the Chinese case and solved by the optimization toolbox of MATLAB. Some insights on investment policies are drawn.

Power production planning. Salahi and Jafari [2016] consider a production planning problem where electricity prices and demands are assumed to be stochastic. A MILP model is formulated, which corresponds to a lot-sizing problem with stochastic components. In this model, the expected profit is maximized, subject to a constraint limiting the CVaR of the profits. A sensitivity analysis on the relevant parameters is performed, and the impact of distinct electricity pricing schemes on the production plan is discussed.

10 Medicine

CVaR has been applied to a specific application arising in medicine, namely, radiation treatment planning for cancer patients. In this problem, a number of radiation beams pass through a patient, depositing energy along their path. Typically, there are several clinical targets to be irradiated, and several nearby organs to be spared. Hence, beams are delivered from a number of different orientations spaced around the patient, so that the intersection of these beams includes the targets. In this way, targets receive the higher radiation dose, whereas the nearby organs receive radiation from one or few beams and are relatively spared. The Fluence Map Optimization (FMO) problem consists in deciding the intensity of each beam so that prescription doses of radiation are delivered to the targets, while tolerance doses of radiation are not exceeded at the nearby organs. The main characteristics of the papers reviewed in this section are summarized in Table VIII.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Romeijn et al. [2006]			√	LP
Romeijn et al. [2005]			√	CP; Column Generation
Chan et al. [2014]			√	Robust LP/SOCP
Ahmed et al. [2010]			√	GH + LP; Param. Search on quantiles

[a] LP: Linear Programming; CP: Convex Programming; SOCP: Second-Order Cone Programming
GH: Greedy Heuristic

Table VIII: References in medicine problems: A summary of the main characteristics.

Radiation treatment planning. Romeijn et al. [2006] note that a natural formulation of radiation treatment planning requires a large number of chance constraints, corresponding to quantiles of tissue units receiving a radiation dose larger (smaller) than the prescription for targets (healthy) tissues. However, these constraints lead to intractable models. Hence, the authors propose to substitute these chance constraints with limits on the average value of doses exceeding given threshold values, resulting in a number of CVaR measures. The consequent formulation is a large-scale LP model, easily manageable with current optimization software packages. The problem of radiation treatment planning is considered by the same authors from a more general viewpoint in Romeijn et al. [2005]. Here, a multileaf collimator is used to assign specific shapes to beam groups. These shapes may change during the treatment. The optimal management of the multileaf collimator results in a difficult problem that includes the FMO considered in the previous work. A convex optimization model is proposed, and a column generation technique proposed. As in the fluence map problem, the prescribed and forbidden doses of radiations are obtained through a number of constraints on appropriate CVaR measures. Chan et al. [2014] consider radiation breast cancer therapy. The problem is similar to the fluence map problem described in Romeijn et al. [2006], but here the position of the targets and the organs is not fixed, due to breath movements. To address this issue, the authors consider a system where a finite number of states (positions) is allowed, and each state is characterized by a known loss distribution. The time spent by the system in a certain state is not known, but described by an ellipsoidal or polyhedral uncertainty set. A weighted sum of losses has to be minimized. The prescription doses are forced by a series of robust-CVaR constraints, where both tail averages for specific loss functions and uncertainty on the loss function to be used are taken into account. The resulting optimization model is tested on clinical data. It is shown that it generalizes two existing clinical methods and shows the advantages of both. Ahmed et al. [2010] propose an automated intensity-modulated radiation therapy planning system based on the FMO model of Romeijn et al. [2006]. Ahmed et al. [2010] include in their analysis measures of quality like the coverage, i.e., the proportion of voxels in the target structure receiving a radiation dose greater than or equal to the prescription dose, and the conformity, i.e., the reciprocal of the proportion of voxels receiving at least the prescribed dose that belongs to the target structure. In their framework, a parametric search is done on the CVaR constraints of the FMO LP model in order to improve, as much as possible, coverage and conformity. Furthermore, a greedy heuristic based on the solution of multiple cardinality constrained knapsack problems is used to improve the beam angle selection. The framework is tested on three case studies, proving the effectiveness of the proposed approach and the superiority with respect to existing approaches.

11 Other topics

In this section, we review those papers dealing with specific applications, and that do not belong to any of the categories listed above. The main characteristics of each paper are first summarized in Table IX.

Reference	Obj.	Weigh. Obj.	Cons.	Add. Details[a]
Gönsch and Hassler [2014]	√			DP + Continuous KP
Van Parys et al. [2016]			√	SDP
Kammerdiner et al. [2014]			√	NLP; Closed-form solutions
Takeda and Kanamori [2009]	√			NLP; SOCP
Hsieh and Lu [2013]	√			IFO; LS + MCS
Fernández [2017]	√			IFO; MCS
Filippi et al. [2017a]	√			MILP

[a] LP: Linear Programming; MILP: Mixed-Integer LP; NLP: Non-LP; CP: Cutting Planes; DP: Dynamic Programming
KP: Knapsack Problem; SDP: Semi-Definite Programming; SOCP: Second-Order Cone Programming
IFO: Integral Function Optimization; LS: Local Search; MCS: Monte Carlo Sampling; MOMILP: Multi-Objective MILP

Table IX: References in miscellaneous problems: A summary of the main characteristics

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Revenue management. Gönsch and Hassler [2014] study the following dynamic revenue management problem. A set of products, with different prices, can be produced, and each unit production requires one unit of a perishable resource, available in a fixed amount. A certain time horizon is divided into microperiods, such that in each microperiod at most one unit of a single product is requested, with a given probability. In each microperiod, we have to decide whether to accept or reject a possible request. Instead of the classic expected revenue objective, the authors consider the CVaR as objective. They develop a dynamic programming procedure, where each decision corresponds to a continuous knapsack problem. A simulation study on a benchmark instance is conducted to show how the new approach may improve the risk profile under various settings.

Optimal control. Van Parys et al. [2016] investigate constrained control problems for stochastic linear systems when only the first two moments of the disturbance distribution are known. The use of distributionally robust chance and CVaR constraints is proposed to express constraint specifications when faced with distributional ambiguity. It is shown that these types of constraint formulations are practically meaningful and computationally tractable in the proposed finite and infinite horizon control design problems. It is shown that the results are completely non-conservative given the information available. The efficacy of the proposed formulation is illustrated for a wind turbine blade control design case study.

Broadcast transmissions. Kammerdiner et al. [2014] consider a transmission problem where a server transmits a stream of packets to a set of clients over a broadcast channel. Initially, each client is in an idle state, and later it switches into an active state. During the idle state, the client cannot receive packets, and the duration of the idle state for each client is a random variable. A loss function is defined, corresponding to the number of clients that do not receive the packets when the broadcast is performed at a certain time. The problem is to find a minimum time for a packet transmission, subject to an upper bound on the CVaR of the loss function. An explicit form for the CVaR constraint is derived when either the idle times are exponentially distributed or they follow a Poisson process. In particular, for the case of a single client, a closed-form expression for the optimum transmission time is derived.

Statistical learning. Takeda and Kanamori [2009] analyze the role of CVaR optimization in statistical learning problems. They develop CVaR minimization models for linear classification and linear regression problems, where input data are subject to measurement errors and are assumed to belong to finite and infinite uncertainty sets. Regarding infinite sets, Monte Carlo sampling is suggested to generate approximate nonlinear optimization models and an upper bound on the approximation error is derived. Takeda and Kanamori [2009] show that several existing statistical learning methods can be regarded as CVaR models built on an appropriate cost function. Extensive computational experiments confirm the theoretical findings.

Acceptance sampling plans. Hsieh and Lu [2013] incorporate the risk-aversion of a decision maker in decisions concerning Bayesian acceptance sampling plans. They consider a censoring scheme where n items are placed in a test until r of them fail. If a function of the total duration time of the r items failed is lower than a threshold c, then the whole lot of items is rejected. Hsieh and Lu [2013] assume a Weibull distribution for an item lifetime, depending on an unknown parameter θ which follows an inverse Gamma distribution. A total cost function of the three unknowns n, r, c is derived, parametrized on θ. The authors formulate a decision problem where the CVaR on the total cost function is minimized. Since the expressions involved are mathematically intractable, an incremental local search algorithm is devised, using Monte Carlo sampling to estimate the objective function values. The results of the proposed model are compared with those of a classic expected cost minimization problem. Fernández [2017] considers the determination of the best inspection scheme for lot acceptance. Briefly speaking, two integer values have to be determined: the cardinality of the lot sample and the minimum total number of defects such that the lot is rejected. The distribution of defects in an item is assumed to follow a Poisson distribution with unknown mean λ. It is also assumed that parameter λ follows a truncated Gamma distribution whose parameters can be updated continuously based on new information. Within these settings, the uncertainty is related to the real value of parameter λ. An appropriate cost function is then defined, and constraints related to acceptable statistical errors are formulated. Hence, the decision problem is formulated as a non-linear integer program in two variables, where the CVaR on the cost is minimized. Since the CVaR cannot be computed explicitly, Monte Carlo sampling is used to derive the approximate optimum values of the decision variable. An example taken from the paper industry is discussed.

Crop planning. Filippi et al. [2017a] consider a crop planning problem where several factors have to be considered simultaneously. These include market price variability of harvested products, specific resource requests for each crop, restrictions caused by limited machine availability, and timing of operations required to complete each crop cultivation. Two different mathematical formulations are provided. The first one looks for the crop-mix that maximizes the farmer's expected profit, as measured by the difference between the revenues obtained by selling the harvested products and the production costs. The second model considers the revenue of each crop as a random variable, since it depends on the price quoted in the exchange market, and the yield per hectare of harvested product. Then, the CVaR of the revenues is maximized. A case study is considered, where the results of the proposed models are compared with the cultivation choice made by a farmer, under different parameter settings. Computational results emphasize the advantage of using the CVaR model for a risk-averse farmer.

12 Concluding remarks

In the domain of financial applications, shortfall or quantile risk measures are receiving an ever increasing attention. The Conditional Value-at-Risk (CVaR) is one of such measures. The range of applicability of CVaR, as also other risk measures, goes far beyond that of optimization in finance. This survey reviews 88 papers, all dating from 2005 or later, where the concept of CVaR is embedded into a decision problem arising in a non-financial context. The related literature is growing ceaselessly. In the Introduction, we have discussed the reasons of this success. Here, we draw a few conclusions from the analysis developed above and identify possible research avenues.

The overview that we provided clearly indicates that some topics, such as inventory and supply chain management, have received a considerable attention from the literature. On the other side, despite their academic and practical relevance, other topics have been studied by very few researchers. In this sense, the most relevant example is represented by scheduling problems with uncertain job processing times, which have been tackled in only 2 papers. Furthermore, given the flexibility of CVaR, it can be applied to several contexts different from those mentioned above. We here only name a few problems that we believe deserve some attention. A first example is related to shortest path or, more generally, routing problems with uncertain traveling costs or times. A second example is related to project management problems when the time needed to complete each task is uncertain. Finally, it might be also worth investigating budget allocation problems where the outcome associate with the allocation is uncertain.

Embedding CVaR into a MILP model using a scenario-based approach implies the introduction of linear inequalities and continuous variables. In principle, this fact should not increase considerably the hardness of the MILP model. Nevertheless, some authors highlight that, to obtain a good approximation of the probability distribution, one requires a large number of scenarios which, in turn, can make the problem too large to be solved exactly within reasonable computing times with a general-purpose solver (see, among others, Boginski et al. [2009], Sorokin et al. [2013], and Mahdavi Pajouh et al. [2017]). Even for medium-size models, some authors noted that the convergence to optimality for a MILP model with CVaR constraints and variables may be much slower than the convergence for the original model (see Filippi et al. [2017b]). That notwithstanding, we are aware of only few papers that analyze the optimization of CVaR from a computational viewpoint. Among these, it is worth mentioning the paper by Künzi-Bay and Mayer [2006] for LP models arising in a financial context and the article by Sarin et al. [2014] regarding MILP models arising in a scheduling context. In both cases, specialized L-shape algorithms are developed. Consequently, the design and analysis of a general cutting plane framework for MILP models with the CVaR is another interesting research topic.

As observed by Sarykalin et al. [2008], the accuracy of CVaR estimation is heavily affected by the accuracy of the model used to estimate the distribution tails. Therefore, if there is not a good model for the tail of the distribution, the CVaR may be misleading. Unfortunately, several authors neglect this issue. In some cases, the authors does not seem to be aware that they are not optimizing the CVaR, but rather an approximation of the CVaR based on a certain set of scenarios. Indeed, the quality of this approximation is usually not considered but, if scenarios are used to approximate CVaR computation, close attention should be paid to how these scenarios are generated.

A few authors apply the CVaR in a deterministic context (see, e.g., Section 10). In this case, CVaR is often used as a measure of fairness, which is essentially the same as k-sum optimization, see Filippi et al. [2017b]. The literature on k-sum optimization is confined to niche applications and algorithmic aspects. The awareness on the strict relation between CVaR optimization and k-sum optimization might renew interest on the latter, considering optimization models of general interest in the context of SCM. In addition, k-sum optimization is defined only for performance measures (scenarios) with the same weight (equiprobable). With general weights, the k-sum concept is not directly applicable, but it should be relaxed to get the equivalent of CVaR for general discrete distributions. The extension of algorithms designed for specific k-sum optimization problems to the weighted case may be a research topic of interest.

Finally, in the financial literature it is customary to use a few values of the parameter α defining the risk-aversion, typically 0.90, 0.95, 0.99. These values are usually adopted also in non-financial applications, but there is no guarantee that in discrete problems such values produce results different from Minimax solutions. Considering that if the CVaR value is optimized, then α close to 1 corresponds to a Minimax approach (highest risk-aversion), whereas α close to 0 corresponds to expected value optimization (risk neutrality), some more efforts should be made to carry out a parametric analysis on the value of α in order to study which compromise solutions better reflect the risk preferences of the decision maker.

References

M.A.M. Abdel-Aal and S.Z. Selim. Risk-averse multi-product selective newsvendor problem with different market entry scenarios under CVaR criterion. *Computers & Industrial Engineering*, 103:250–261, 2017.

C. Acerbi. Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26:1505–1518, 2002.

S. Ahmed, U. Çakmak, and A. Shapiro. Coherent risk measures in inventory problems. *European Journal of Operational Research*, 182:226–238, 2007.

S. Ahmed, O. Gozbasi, M. Savelsbergh, I. Crocker, T. Fox, and E. Schreibmann. An automated intensity-modulated radiation therapy planning system. *INFORMS Journal on Computing*, 22:568–583, 2010.

A.H. Ansaripoor, F.S. Oliveira, and A. Liret. A risk management system for sustainable fleet replacement. *European Journal of Operational Research*, 237:701–712, 2014.

A.H. Ansaripoor, F.S. Oliveira, and A. Liret. Recursive expected Conditional Value at Risk in the fleet renewal problem with alternative fuel vehicles. *Transportation Research Part C: Emerging Technologies*, 65:156–171, 2016.

E. Arıkan and J. Fichtinger. The risk-averse newsvendor problem under spectral risk measures: A classification with extensions. *European Journal of Operational Research*, 256:116–125, 2017.

P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9: 203–228, 1999.

V.L. Boginski, C.W. Commander, and T. Turko. Polynomial-time identification of robust network flows under uncertain arc failures. *Optimization Letters*, 3:461–473, 2009.

E. Borgonovo and L. Peccati. Financial management in inventory problems: Risk averse vs risk neutral policies. *International Journal of Production Economics*, 118:233–242, 2009.

S.V.B. Bruno and C. Sagastizábal. Optimization of real asset portfolio using a coherent risk measure: Application to oil and energy industries. *Optimization and Engineering*, 12:257–275, 2011.

O. Caliskan-Demirag, Y.F. Chen, and J. Li. Customer and retailer rebates under risk aversion. *International Journal of Production Economics*, 133:736–750, 2011.

M.C. Carneiro, G.P. Ribas, and S. Hamacher. Risk management in the oil supply chain: A CVaR approach. *Industrial & Engineering Chemistry Research*, 49:3286–3294, 2010.

J.P.S. Catalão, H.M.I. Pousinho, and J. Contreras. Optimal hydro scheduling and offering strategies considering price uncertainty and risk management. *Energy*, 37:237–244, 2012.

K. Chahar and K. Taaffe. Risk averse demand selection with all-or-nothing orders. *Omega*, 37:996–1006, 2009. doi: 10.1287/opre.1080.0603. URL `http://dx.doi.org/10.1287/opre.1080.0603`.

T.C.Y. Chan, H. Mahmoudzadeh, and T.G. Purdie. A robust-CVaR optimization approach with application to breast cancer therapy. *European Journal of Operational Research*, 238:876–885, 2014.

Z. Chang, S. Song, Y. Zhang, J.-Y. Ding, R. Zhang, and R. Chiong. Distributionally robust single machine scheduling with risk aversion. *European Journal of Operational Research*, 256:261–274, 2017. doi: DOI 10.1016/j.ejor.2016.06.025.

G. Chen, M.S. Daskin, Z.-J.M. Shen, and S. Uryasev. The α-reliable mean-excess regret model for stochastic facility location modeling. *Naval Research Logistics*, 53:617–626, 2006.

Y.(F.) Chen, M. Xu, and Z.G. Zhang. A risk-averse newsvendor model under the CVaR criterion. *Operations Research*, 57:1040–1044, 2009. doi: 10.1287/opre.1080.0603. URL `http://dx.doi.org/10.1287/opre.1080.0603`.

L. Cheng, Z. Wan, and G. Wang. Bilevel newsvendor models considering retailer with CVaR objective. *Computers & Industrial Engineering*, 57:310–318, 2009.

J. Claro and J.P. de Sousa. A multiobjective metaheuristic for a mean-risk multistage capacity investment problem. *Journal of Heuristics*, 16:85–115, 2010.

J. Claro and J.P. de Sousa. A multiobjective metaheuristic for a mean-risk multistage capacity investment problem with process flexibility. *Computers & Operations Research*, 39:838–849, 2012.

C.W. Commander, P.M. Pardalos, V. Ryabchenko, S. Uryasev, and G. Zrazhevsky. The wireless network jamming problem. *Journal of Combinatorial Optimization*, 14:481–498, 2007.

J. Doege, P. Schiltknecht, and H.-J. Lüthi. Risk management of power portfolios and valuation of flexibility. *OR Spectrum*, 28:267–287, 2006.

G.D. Eppen, R.K. Martin, and L. Schrage. A scenario approach to capacity planning. *Operations Research*, 37:517–527, 1989.

S. Faghih-Roohi, Y.-S. Ong, S. Asian, and A.N. Zhang. Dynamic Conditional Value-at-Risk model for routing and scheduling of hazardous material transportation networks. *Annals of Operations Research*, 247:715–734, 2016. doi: 10.1007/s10479-015-1909-2.

A.J. Fernández. Economic lot sampling inspection from defect counts with minimum Conditional Value-at-Risk. *European Journal of Operational Research*, 258:573âĂŞ–580, 2017.

C. Filippi, R. Mansini, and E. Stevanato. Mixed integer linear programming models for optimal crop selection. *Computers & Operations Research*, 81:26–39, 2017a.

C. Filippi, W. Ogryczak, and M.G. Speranza. Beyond average and minimax in MILP. Technical report, University of Brescia, April 2017b.

J. García-González, E. Parrilla, and A. Mateo. Risk-averse profit-based optimal scheduling of a hydro-chain in the day-ahead electricity market. *European Journal of Operational Research*, 181:1354–1369, 2007.

A. Givler Chapman and J.E. Mitchell. A fair division approach to humanitarian logistics inspired by Conditional Value-at-Risk. *Annals of Operations Research*, Forthcoming. doi: 10.1007/s10479-016-2322-1.

M. Goh and F. Meng. A stochastic model for supply chain risk management using Conditional Value at Risk. In T. Wu and J. Blackhurst, editors, *Managing Supply Chain Risk and Vulnerability*, pages 141–157. Springer, 2009.

J. Gönsch and M. Hassler. Optimizing the Conditional Value-at-Risk in revenue management. *Review of Managerial Science*, 8:495–521, 2014. ISSN 1863-6691. doi: 10.1007/s11846-013-0114-4. URL http://dx.doi.org/10.1007/s11846-013-0114-4.

J.-y. Gotoh and Y. Takano. Newsvendor solutions via Conditional Value-at-Risk minimization. *European Journal of Operational Research*, 179:80–96, 2007.

A.R. Hatami, H. Seifi, and M.K. Sheikh-El-Eslami. Optimal selling price and energy procurement strategies for a retailer in an electricity market. *Electric Power Systems Research*, 79:246–254, 2009.

I. Heckmann, T. Comes, and S. Nickel. A critical review on supply chain risk – definition, measure and modeling. *Omega*, 52:119–132, 2015.

R. Hemmati, H. Saboori, and S. Saboori. Stochastic risk-averse coordinated scheduling of grid integrated energy storage units in transmission constrained wind-thermal systems within a Conditional Value-at-Risk framework. *Energy*, 113:762–775, 2016. ISSN 0360-5442. doi: http://dx.doi.org/10.1016/j.energy.2016.07.089. URL http://www.sciencedirect.com/science/article/pii/S0360544216310143.

M. Hosseini-Firouz. Optimal offering strategy considering the risk management for wind power producers in electricity market. *International Journal of Electrical Power & Energy Systems*, 49:359–368, 2013.

C.-C. Hsieh and Y.-T. Lu. Manufacturer's return policy in a two-stage supply chain with two risk-averse retailers and random demand. *European Journal of Operational Research*, 207:514–523, 2010.

C.-C. Hsieh and Y.-T. Lu. Risk-embedded bayesian acceptance sampling plans via Conditional Value-at-Risk with Type II censoring. *Computers & Industrial Engineering*, 65:551–560, 2013.

W. Jammernegg and P. Kischka. Risk-averse and risk-taking newsvendors: a Conditional Expected Value approach. *Review of Managerial Science*, 1:93–110, 2007.

W. Jammernegg and P. Kischka. Risk preferences and robust inventory decisions. *International Journal of Production Economics*, 118:269–274, 2009.

W. Jammernegg and P. Kischka. Risk preferences of a newsvendor with service and loss constraints. *International Journal of Production Economics*, 143:410–415, 2013.

K. Kalinchenko, A. Veremyev, V. Boginski, D.E. Jeffcoat, and S. Uryasev. Robust connectivity issues in dynamic sensor networks for area surveillance under uncertainty. *Pacific Journal of Optimization*, 7: 235–248, 2011.

A. Kammerdiner, A. Sprintson, E. Pasiliao, and V. Boginski. Optimization of discrete broadcast under uncertainty using Conditional Value-at-Risk. *Optimization Letters*, 8:45–59, 2014. ISSN 1862-4480. doi: 10.1007/s11590-012-0542-0. URL http://dx.doi.org/10.1007/s11590-012-0542-0.

A.P. Katariya, S. Çetinkaya, and E. Tekin. On the comparison of risk-neutral and risk-averse newsvendor problems. *Journal of the Operational Research Society*, 65:1090–1107, 2014.

P. Krokhmal, J. Palmquist, and S. Uryasev. Portfolio optimization with Conditional Value-at-Risk objective and constraints. *Journal of Risk*, 4:43–68, 2002.

A. Künzi-Bay and J. Mayer. Computational aspects of minimizing Conditional Value-at-Risk. *Computational Management Science*, 3:3–27, 2006.

Z. Lu, J. Qi, B. Wen, and X. Li. A dynamic model for generation expansion planning based on Conditional Value-at-Risk theory under low-carbon economy. *Electric Power Systems Research*, 141:363–371, 2016.

J. Ma, F.M. Pajouh, B. Balasundaram, and V. Boginski. The minimum spanning $k-$core problem with bounded CVaR under probabilistic edge failures. *INFORMS Journal on Computing*, 28:295–307, 2016.

L. Ma, F. Liu, S. Li, and H. Yan. Channel bargaining with risk-averse retailer. *International Journal of Production Economics*, 139:155–167, 2012.

F. Mahdavi Pajouh, E. Moradi, and B. Balasundaram. Detecting large risk-averse 2-clubs in graphs with random edge failures. *Annals of Operations Research*, 249:55–73, 2017.

H. Markowitz. Portfolio selection. *The Journal of Finance*, 7:77–91, 1952.

R. Mena, M. Hennebel, Y.-F. Li, and E. Zio. A multi-objective optimization framework for risk-controlled integration of renewable generation into electric power systems. *Energy*, 106:712–727, 2016.

I.G. Moghaddam, M. Nick, F. Fallahi, M. Sanei, and S. Mortazavi. Risk-averse profit-based optimal operation strategy of a combined wind farm-cascade hydro system in an electricity market. *Renewable Energy*, 55:252–259, 2013.

N. Noyan. Risk-averse two-stage stochastic programming with an application to disaster management. *Computers & Operations Research*, 39:541–559, 2012.

W. Ogryczak and A. Ruszczyński. Dual stochastic dominance and related mean-risk models. *SIAM Journal on Optimization*, 13:60–78, 2002.

W. Ogryczak, H. Luss, M. Pióro, D. Nace, and A. Tomaszewski. Fair optimization and networks: A survey. *Journal of Applied Mathematics*, 2014:1–25, 2014.

G.C. Pflug. Some remarks on the Value-at-Risk and the Conditional Value-at-Risk. In S. Uryasev, editor, *Probabilistic Constrained Optimization: Methodology and Applications*, pages 272–281. Springer US, 2000.

H.M.I. Pousinho, V.M.F. Mendes, and J.P.S. Catalão. A risk-averse optimization model for trading wind energy in a market environment under uncertainty. *Energy*, 36:4935–4942, 2011.

Y. Qin, R. Wang, A.J. Vakharia, Y. Chen, and M.M.H. Seref. The newsvendor problem: Review and directions for future research. *European Journal of Operational Research*, 213:361–374, 2011.

R. Qiu, J. Shang, and X. Huang. Robust inventory decision under distribution uncertainty: A CVaR-based optimization approach. *International Journal of Production Economics*, 153:13–23, 2014.

R.T. Rockafellar and S. Uryasev. Optimization of Conditional Value-at-Risk. *Journal of Risk*, 2:21–42, 2000.

R.T. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general loss distributions. *Journal of Banking & Finance*, 26:1443–1471, 2002.

R.T. Rockafellar, S. Uryasev, and M. Zabarankin. Generalized deviations in risk analysis. *Finance and Stochastics*, 10:51–74, 2006.

H.E. Romeijn, R.K. Ahuja, J.F. Dempsey, and A. Kumar. A column generation approach to radiation therapy treatment planning using aperture modulation. *SIAM Journal on Optimization*, 15:838–862, 2005.

H.E. Romeijn, R.K. Ahuja, J.F. Dempsey, and A. Kumar. A new linear programming approach to radiation therapy treatment planning problems. *Operations Research*, 54:201–216, 2006.

M. Rysz, M. Mirghorbani, P. Krokhmal, and E.L. Pasiliao. On risk-averse maximum weighted subgraph problems. *Journal of Combinatorial Optimization*, 28:167–185, 2014.

N. Salahi and M.A. Jafari. Energy-performance as a driver for optimal production planning. *Applied Energy*, 174:88–100, 2016.

S.C. Sarin, H.D. Sherali, and L. Liao. Minimizing Conditional-Value-at-Risk for stochastic scheduling problems. *Journal of Scheduling*, 17:5–15, 2014.

S. Sarykalin, G. Serraino, and S. Uryasev. Value-at-Risk vs. Conditional Value-at-Risk in risk management and optimization. In Z.-L. Chen and S. Raghavan, editors, *Tutorials in Operations Research*, pages 270–294. INFORMS, 2008.

T. Sawik. Selection of supply portfolio under disruption risks. *Omega*, 39:194–208, 2011a.

T. Sawik. Supplier selection in make-to-order environment with risks. *Mathematical and Computer Modelling*, 53:1670–1679, 2011b.

T. Sawik. Integrated selection of suppliers and scheduling of customer orders in the presence of supply chain disruption risks. *International Journal of Production Research*, 51:7006–7022, 2013a.

T. Sawik. Selection of resilient supply portfolio under disruption risks. *Omega*, 41:259–269, 2013b.

T. Sawik. Optimization of cost and service level in the presence of supply chain disruption risks: Single vs. multiple sourcing. *Computers & Operations Research*, 51:11–20, 2014.

S.R. Seyedshohadaie, I. Damnjanovic, and S. Butenko. Risk-based maintenance and rehabilitation decisions for transportation infrastructure networks. *Transportation Research Part A: Policy and Practice*, 44:236–248, 2010.

H. Soleimani and K. Govindan. Reverse logistics network design and planning utilizing Conditional Value at Risk. *European Journal of Operational Research*, 237:487–497, 2014.

H. Soleimani, M. Seyyed-Esfahani, and G. Kannan. Incorporating risk measures in closed-loop supply chain network design. *International Journal of Production Research*, 52:1843–1867, 2014.

A. Sorokin, V. Boginski, A. Nahapetyan, and P.M. Pardalos. Computational risk management techniques for fixed charge network flow problems with uncertain arc failures. *Journal of Combinatorial Optimization*, 25:99–122, 2013.

Q. Sun, Y. Dong, and W. Xu. Effects of higher order moments on the newsvendor problem. *International Journal of Production Economics*, 146:167–177, 2013.

M.A. Tajeddini, A. Rahimi-Kian, and A. Soroudi. Risk averse optimal operation of a virtual power plant using two stage stochastic programming. *Energy*, 73:958–967, 2014.

A. Takeda and T. Kanamori. A robust approach based on Conditional Value-at-Risk measure to statistical learning problems. *European Journal of Operational Research*, 198:287–296, 2009.

B. Tomlin and Y. Wang. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*, 7:37–57, 2005.

I. Toumazis and C. Kwon. Routing hazardous materials on time-dependent networks using Conditional Value-at-Risk. *Transportation Research Part C: Emerging Technologies*, 37:73–92, 2013.

I. Toumazis and C. Kwon. Worst-case Conditional Value-at-Risk minimization for hazardous materials transportation. *Transportation Science*, 50:1174–1187, 2016.

I. Toumazis, C. Kwon, and R. Batta. Value-at-Risk and Conditional Value-at-Risk minimization for hazardous materials routing. In R. Batta and C. Kwon, editors, *Handbook of OR/MS Models in Hazardous Materials Transportation*, pages 127–154. Springer, 2013.

B.P.G. Van Parys, D. Kuhn, P.J. Goulart, and M. Morari. Distributionally robust control of constrained stochastic systems. *IEEE Transactions on Automatic Control*, 61:430–442, 2016.

J. Wu, S. Wang, X. Chao, C.T. Ng, and T.C.E. Cheng. Impact of risk aversion on optimal decisions in supply contracts. *International Journal of Production Economics*, 128:569–576, 2010.

M. Wu, S.X. Zhu, and R.H. Teunter. The risk-averse newsvendor problem with random capacity. *European Journal of Operational Research*, 231:328–336, 2013a.

M. Wu, S.X. Zhu, and R.H. Teunter. Newsvendor problem with random shortage cost under a risk criterion. *International Journal of Production Economics*, 145:790–798, 2013b.

M. Wu, S.X. Zhu, and R.H. Teunter. A risk-averse competitive newsvendor problem under the CVaR criterion. *International Journal of Production Economics*, 156:13–23, 2014.

X. Xinsheng, M. Zhiqing, S. Rui, J. Min, and J. Ping. Optimal decisions for the loss-averse newsvendor problem under CVaR. *International Journal of Production Economics*, 164:146–159, 2015.

M. Xu. A price-setting newsvendor model under CVaR decision criterion with emergency procurement. *Journal of Systems Science and Systems Engineering*, 19:85–104, 2010.

M. Xu and J. Li. Optimal decisions when balancing expected profit and Conditional Value-at-Risk in newsvendor models. *Journal of Systems Science and Complexity*, 23:1054–1070, 2010.

X. Xu, Z. Meng, and R. Shen. A tri-level programming model based on Conditional Value-at-Risk for three-stage supply chain management. *Computers & Industrial Engineering*, 66:470–475, 2013.

L. Yang, M. Xu, G. Yu, and H. Zhang. Supply chain coordination with CVaR criterion. *Asia-Pacific Journal of Operational Research*, 26:135–160, 2009.

S. Yau, R.H. Kwon, J.S. Rogers, and D. Wu. Financial and operational decisions in the electricity sector: Contract portfolio optimization with the Conditional Value-at-Risk criterion. *International Journal of Production Economics*, 134:67–77, 2011.

O. Yezerska, S. Butenko, and V.L. Boginski. Detecting robust cliques in graphs subject to uncertain edge failures. *Annals of Operations Research*, Forthcoming. doi: 10.1007/s10479-016-2161-0.

Y. Yin. Robust optimal traffic signal timing. *Transportation Research Part B: Methodological*, 42:911–924, 2008.

D. Zhang, H. Xu, and Y. Wu. Single and multi-period optimal inventory control models with risk-averse constraints. *European Journal of Operational Research*, 199:420–434, 2009.

L. Zhang and Y. Yin. Robust synchronization of actuated signals on arterials. *Transportation Research Record: Journal of the Transportation Research Board*, 2080:111–119, 2008.

Q.P. Zheng and P.M. Pardalos. Stochastic and risk management models and solution algorithm for natural gas transmission network expansion and LNG terminal location planning. *Journal of Optimization Theory and Applications*, 147:337–357, 2010.

Y.-j. Zhou, X.-h. Chen, and Z.-r. Wang. Optimal ordering quantities for multi-products with stochastic demand: Return-CVaR model. *International Journal of Production Economics*, 112:782–795, 2008.