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Recent results on the wave equations with local Kelvin-Voigt damping

Abstract

We consider a one-dimensional wave equation with local Kelvin-Voigt damping

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left( a \frac{\partial u}{\partial x} + d(x) \frac{\partial^2 u}{\partial t \partial x} \right), \quad x \in (-1, 1), \ t > 0$$

with certain boundary and initial conditions. The damping coefficient $d(x)$ has support of a subinterval in $(-1, 1)$. Hence, the damping is local. In 1999, it was found that when $d(x)$ is the characteristic function of a subinterval, the energy of the system does not decay exponentially (i.e., the system is not exponentially stable). This is unexpected since the viscous damping does not have the same behavior. Meanwhile, this also make the problem interesting. In this talk, we will review the existing results and present our recent advances in this direction.