Department of Mathematics
Engineering Faculty
University of Brescia

Research groups

- Algebra and Geometry
- Mathematical Analysis
- Mathematical Physics
- Numerical Analysis
Algebra and Geometry group

Members of the research group

Prof. Silvia PELLEGRINI
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Major Research Topics

1. Cryptography and error correcting codes
   Combinatorial structures might be used to construct error correcting codes and cryptographic schemes. We are interested in obtaining good codes from noteworthy geometrical objects embedded in a finite plane. This topic of research also encompasses application of algebraic techniques to efficient design and implementation of cryptosystems.

2. Combinatorial structures in finite planes
   We investigate structures embedded in finite projective planes and strive to provide characterisations based upon their combinatorial or group-theoretic properties. This is of relevant theoretical interest, as it provides insights on the geometries involved, but presents also practical applications.

3. Graph decompositions
   This research is aimed at determining regular decompositions of the complete graph and investigating their properties. This topic is part of design theory. More in particular, we are interested in both determining new classes of decompositions for a given graph and in constructing functions, like down-links and metamorphoses, linking different, yet closely related, graph designs.

4. Linear spaces with parallelism
   We are interested in investigating linear spaces (not necessarily projective) endowed with one or more parallelism relations among the lines. In particular we are dealing with the study of automorphisms groups of 3-dimensional projective spaces on fields of arbitrary characteristic equipped with the two Clifford parallelisms, as introduced and studied in earlier papers.

5. Loops and geometries
   We are working on problems related to the algebrization of geometric structures through loops, in particular we aim at building new loops starting from suitable sets of collineations of the hyperbolic plane. More generally, we are interested in
building loops exploiting regular permutation sets and in inspecting their properties in relations with those of the starting permutation set.

6. **Nearrings**

Algebraic properties of classes of finite nearrings (weakly divisible nearrings, circular planar nearrings) are studied, in order to determine combinatorial properties to be used to construct balanced or partially balanced designs and the related codes.

**Mathematical Analysis group**

**Members of the research group**

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Dr. Paola TREBESCHI  
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**Major Research Topics**

1. **Nonlinear Hyperbolic Equations in Fluid Dynamics**

The research concerns the analysis of some free boundary problems for multi-dimensional nonlinear equations of hyperbolic type arising in Fluid Dynamics or Magneto-Hydro Dynamics (MHD), when the dissipative effect of viscosity or magnetic resistivity is negligible. Our analysis concern problems as the contact discontinuities (vortex sheets) for the Euler equations of motion of inviscid compressible fluids, the plasma-vacuum interface model (interesting for instance for magnetic confinement) or current-vortex sheets in MHD. The interest for the latter is motivated from Astrophysics as a model for the Kelvin-Helmholtz instability on Saturn's atmosphere (Courtesy by NASA)
heliopause, the outer "boundary" of the solar system. Our research regards the study of existence of solutions in suitable function spaces and their stability and qualitative properties. A second direction of research is concerned with the study of some approximate regularizing models of the standard MHD equations, known as MHD-alpha models. These models were introduced in the literature, because of the impossibility to handle directly, neither analytically nor via direct numerical simulations, the standard MHD system used to compute the turbulent behavior of incompressible magnetofluids. Several MHD-alpha models, in 2D or 3D, have been considered in the viscous, partial viscous or ideal cases.

2. **Characteristic Hyperbolic Problems**

The research is concerned with the study of general mixed initial-boundary value problems for linear hyperbolic equations, when the boundary of the spatial domain is characteristic. Under an assumption of strong or weak well posedness of the problem, the regularity of solutions, in the framework of suitable weighted anisotropic spaces of Sobolev type, has been studied.

3. **Hyperbolic systems of Conservation Laws and Applications**

The focus of the research is on the analysis of Hyperbolic Systems of Conservation Laws, their basic theory, their applications and their optimal control. Particular attention is devoted to the study of models concerning Crowd Dynamics, Traffic Flow, Gas Dynamics, Granular Matters, Phase Transition and Combustion.

4. **Calculus of Variations**

The focus of the research is on the application of methods of the Calculus of Variations to problems arising in Continuum Mechanics, in particular in Fracture Mechanics and Plasticity. Concerning Fracture Mechanics, special attention has been devoted to the model of quasistatic crack propagation proposed by Francfort and Marigo (J. Mech. Phys. Solids 1998). Concerning application to Plasticity, the focus has been on strain gradient plasticity models, in connection to quasistatic evolutions. The mathematical framework involved is that of the "energetic approach" to rate independent evolutions formalized by Mielke and his school (Hand. Differ. Equ. 2005).
5. **Models for phase transitions, damage, and adhesive contact**

Various models describing phase change phenomena (in the reversible and irreversible cases) caused by microscopic movements, as well as damaging in elastic bodies, and contact with adhesion between solids, have been analyzed. For the associated evolutionary PDE systems, of parabolic nature, existence, uniqueness and regularity of solutions have been proved. The long-time behaviour of solutions has also been studied.

6. **Abstract Evolution Equations in Banach and Metric Spaces**

The focus is on evolution equations set in Banach and metric spaces. In particular, gradient flows in Hilbert spaces with nonconvex energies have been studied. Moreover, existence and approximation results for doubly nonlinear equations in metric spaces and in Banach spaces (in the rate-independent case) have been obtained.

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**Mathematical Physics group**

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**Members of the research group**

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Prof. Maria Grazia NASO  
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Prof. Elena VUK  
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**Major Research Topics**

On account of the past experiences and competences, the main interest of the Group is addressed to the modeling of complex systems (physical, chemical, and biological) and materials of interest in applications, jointly with the study of integral and integrodifferential evolution equations governing their evolution. The main goals of the Group will be concerned with the mathematical modeling, controllability of solutions and analysis of contact and transmission problems in continuous media whose behavior is ruled by constitutive equations with memory (thermo-viscoelastic solids and fluids, hereditary heat conductors, ...) and/or exhibiting phase transition phenomena. In the framework of the national project, the members of this Group have specific knowledge to study:

- compatibility of the model within the framework of non-classical thermodynamic theories (extended and irreversible thermodynamics, energy and entropy extra-flux assumptions, ...),
- analysis of well-posedness, control and asymptotic behavior (in time) of solutions to nonlinear PDEs and IPDEs arising in the modeling.

1. **Phase transition phenomena**
   a. **Modeling**
      i. A phase-field model for non isochoric phase separation induced both by temperature and pressure.
ii. Non isothermal, anisotropic, phase-field models describing spontaneous magnetization in the paramagnetic-ferromagnetic transition.

iii. Non isothermal phase-field models describing the isotropic-nematic transition in liquid crystals.


b. Direct and control problems


ii. Well-posedness results and longtime behaviour of solutions for singular phase-field models in which the standard internal energy balance is replaced by an entropy balance with memory or more general laws.

iii. Analysis of well-posedness and global longtime behavior in the history space setting of a phase-field model with thermal memory (Coleman-Gurtin heat flux law) and a third order nonlinearity in the latent heat.

iv. Well-posedness and stability of solutions to a nonlinear system describing the non-isothermal, paramagnetic-ferromagnetic (vector) transition and involving a singular potential of the log type (see 1.a.ii).

v. Boundary controllability of solutions to a simple phase-field model with thermal memory (in the heat flux) describing a first order transition.

2. Viscoelastic models

a. Modeling

i. Thermo-viscoelastic beams and plates fixed at the boundary and involving a kinematic nonlinear term accounting for their extensibility (Woinovsky-Krieger, Berger, etc.) with different constitutive laws for the heat flux (Fourier, Coleman-Gurtin and Gurtin-Pipkin).

ii. Coupled suspension bridge equations when both the main cable and the road bed are composed of a viscoelastic material.

iii. Some viscoelastic models with nonlinear memory (cubic in some sense) having a special class of fourth-order free energy functionals.

b. Thermo-viscoelastic models: direct problems

i. Longtime behavior and existence of the global attractor for the thermo-viscoelastic extensible beam (cf. the item 2.a.i).
ii. Steady states and longtime dynamics for an extensible elastic beam on a viscoelastic foundation, and for an extensible thermoelastic beam on an elastic foundation.

iii. Stationary solutions and well-posedness of the IBVP for the doubly nonlinear suspension bridge equation obtained from model 2.a.ii by keeping the main cable fixed.

iv. Control on the boundary for transmission problem in composite (elastic and thermo-viscoelastic) materials.

v. Determination of constitutive parameters in a (elastic/viscoelastic/thermo-viscoelastic) multilayer through data associated with the reflected wave in a reflection-transmission process.

**Numerical Analysis group**

**Members of the research group**

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Prof. Paola GERVASIO  
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**Major Research Topics**

The research activity of the Numerical Analysis Group is oriented towards the approximation of Partial Differential Equations (PDE) and contributes by developing new computational methodologies based, in different ways, on the interaction of various differential models and/or of several numerical discretization methods.

**Fluid-structure interaction**

The nonlinear coupling of the equations governing fluid-structure interaction systems requires appropriate numerical approaches in order to deal with the motion of the domains occupied by solid and fluid. One of the main issues is the construction of stable numerical schemes. Two different approaches have been considered: (i) Immersed boundary method, (ii) Arbitrary Lagrangian-Eulerian formulation.
1. **Approximation of PDEs by finite element methods**
   The finite element method is one of the most popular methods available for the numerical resolution of PDEs of different types. In view of practical applications, the finite elements methods need to be robust, efficient and accurate. In the case of finite element for problems in mixed form, this requires that some compatibility conditions are satisfied.

   i. Finite element methods for the approximation of eigenproblem in mixed form
   
   ii. Finite element approximation of evolution problem in mixed form
   
   iii. Edge finite elements for Maxwell and photonic crystal equations
   
   iv. Finite elements for the Stokes problem

2. **Domain Decomposition Methods for Heterogeneous Problems**
   Subdomain splitting is an interesting path towards multiphysics, i.e. the use of mathematical models based on different kind of PDE to address physical problems of heterogeneous nature in different subregions of a given computational domain.

   i. Heterogeneous Domain Decompositions by Virtual Control Methods
   
   ii. Extended Variational Formulation for Heterogenous PDE's

3. **High-order methods for the approximation of PDE's**
   Spectral Methods are high order methods for solving PDE's which offer the best performance (in terms of computational efficiency and in handling complex geometries) when they are coupled with either low-order methods (such as finite elements) inside the preconditioning step, and domain decomposition techniques. Algebraic Fractional-Step Schemes are very efficient and accurate techniques to approximate time-dependent PDE's as, e.g., the incompressible Navier-Stokes equations.

   i. Finite-element preconditioning of spectral methods
   
   ii. Algebraic fractional step schemes for the incompressible Navier-Stokes equations

![Navier-Stokes simulation (P. Gervasio)](image-url)